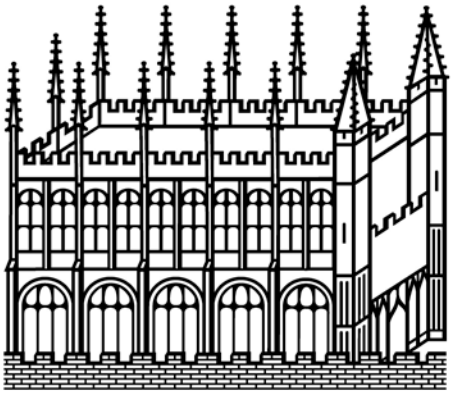


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# DUBLIN PROBLEMS:

BEING

## A COLLECTION OF QUESTIONS

PROPOSED TO THE CANDIDATES

FOR

THE GOLD MEDAL,

AT THE

## GENERAL EXAMINATIONS,

FROM 1816 TO 1822 INCLUSIVE.

WHICH IS SUCCEEDED BY AN ACCOUNT OF THE FELLOWSHIP EXAMINATION, IN 1823.

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LONDON:

G. AND W. B. WHITTAKER,

AVE-MARIA-LANE.

1823.

LONDON:

PRINTED BY R. GILBERT,

ST. JOHN'S SQUARE.



# UNIVERSITY OF DUBLIN,

## 1823.

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*A* List *of the successful Candidates for the Gold Medal, from* 1816 *to* 1822 *inclusive.*

| | | |
|---|---|---|
| 1816 | April | Mr. Corballis. |
| | October | Mr. Hayes. |
| 1817 | April | Mr. Stevelly. |
| | October | Mr. Stack.—(Since Fellow.) |
| 1818 | April | Mr. Moore. |
| | October | Mr. Boyton.—(Since Fellow.) |
| 1819 | April | Mr. Hassard. |
| | October | Mr. Lloyd. |
| 1820 | April | Mr. Haig. |
| | October | Mr. Luby. |
| 1821 | April | Mr. Johnson. |
| 1822 | January | Mr. Twigg. |
| | April | Mr. Lynch. |
| | October | Mr. Longfield. |

# DUBLIN

# Problems.

## 1816.

## APRIL.

### FIRST MORNING.

#### TRIGONOMETRY.

1. If two right angled spherical triangles have a common side, the sines of the hypothenuses are inversely as the sines of the angles opposite to the common side.

2. Solve the equation,

$$x^3 + px - q = 0$$

by the angle the tangent of which is $\frac{2p^{\frac{3}{2}}}{3^{\frac{3}{2}}q}$.

3. If $a$, $b$, $c$, be the sides, and $A$, $B$, $C$, the opposite angles of a spherical triangle,

$$\sin. b = \tan. a \,(\sin. C \cot. A + \cos. C \cos. b.)$$

4. In a spherical triangle, given the base, the sum of the sides, and the sum of the base angles, to determine the triangle.

5. Given two sides of a spherical triangle, when is its area a maximum?

6. If two right angled spherical triangles have

a common side, the cosines of the hypothenuses are proportional to the cosines of the sides not common.

7. In a spherical triangle given a side and an adjacent angle, to express the ratio of the simultaneous increments of the sides.

8. Given two sides of a spherical triangle, to find when a given error in observing one of the base angles, shall produce the least possible error in computing the other.

9. In right angled spherical triangles, having a common hypothenuse, the tangents of the acute angles are reciprocally proportional.

10. Given the hypothenuse of a right angled spherical triangle, to construct it so that the sum of the cosines of the sides shall be a minimum.

11. Find the sum of

$$\frac{1}{1^4}+\frac{1}{3^4}+\frac{1}{5^4}+\frac{1}{7^4}\cdots\cdots$$

continued in infinitum.

12. What is the sum of the products of every side of a plane polygon, multiplied by the cosine of the angle, at which it is inclined to a given line, in the plane of the polygon; the angles being all supposed to be measured in the same direction?

13. If $\pi$ represent a semicircle, determine the sine, cosine, &c. of $\frac{4n+1}{2}\pi$; $n$ being supposed a positive integer.

14. Give a general formula representing all arcs whose sines are negative.

15. Prove the formulæ,

$$\sin.(n \pm 1)\,a = \sin. n\,a \cos. a \pm \sin. a \cos. n\,a.$$

16. The sum of the sines of the doubles of two angles is equal to twice the product of the sine of the sum, into the cosine of the difference.

17. If $s$ be the side, and $b$ the base of a plane isosceles triangle, and $\varphi$ the vertical angle,

$$\cos. \varphi = \frac{2\,s^2 - b^2}{2\,s^2}.$$

18. If the sum of the sides of a spherical triangle, and the included angle, be given, the tangent of half the sum of the base angles is proportional to the cosine of half the difference of the sides.

19. If the difference of the sides of a spherical triangle, and the included angle, be given, the tangent of half the difference of the base angles varies inversely as the sine of half the sum of the sides.

20. If the difference of two sides of a spherical triangle, and the included angle, be given, the sine of half the difference of the base angles bears an invariable ratio to the sine of half their sum.

21. If a given angle be divided into any two parts, the difference of the sines of those parts bears a given ratio to the difference of their cosines.

22. To express the area of a plane triangle as a function of two sides, and the included angle.

23. To divide a given angle into two parts, so that the sum of their tangents may be a minimum.

24. The sine of the side of a right angled spherical triangle is equal to half the difference of the cosines of the sum and difference of the hypothenuse and opposite angle.

25. The cosine of the hypothenuse of a right angled spherical triangle is equal to half the sum of the cosines of the sum and difference of the sides.

26. In a spherical right angled triangle, given an acute angle, and the sum of the opposite side and the remaining acute angle, to determine the triangle.

27. In a right angled spherical triangle, the tangent of half of one of the sides is a geometric mean between the tangents of half the sum and half the difference of the other sides and the hypothenuse.

28. In a right angled spherical triangle, the sine of the sum of the hypothenuse and one side, is to the sine of the difference, as the square of the tangent of half the included angle is to the radius.

29. In every right angled spherical triangle, the difference of the two oblique angles is less than 90°.

30. In every spherical triangle, if all the angles are acute, each of the sides is less than 90°.

## FIRST EVENING.

### GEOMETRY.

1. Given the three sides of a triangle, to express its area.

2. Adapt this expression to logarithmic calculation.

3. Shew that the method of finding the area in Fifth Proposition, Second Book of Elrington's Euclid, leads to the same formula.

4. What are the peculiar advantages of the algebraic method?

5. Given the equation of a straight line, to determine the equation of a straight line perpendicular to it passing through a given point.

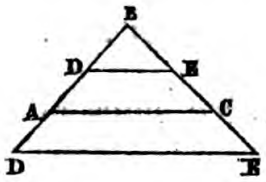
6. To find a formula expressing the length of a perpendicular from a given point upon a given straight line, as a function of the co-ordinates of the point and the constants of the equation of the right line.

7. Shew how this formula may be established from geometrical and trigonometrical principles.

8. A triangle being drawn in the plane of a system of two rectangular axes of co-ordinates, to express its sides and area as functions of the co-ordinates of its three vertices.

9. A triangle, $ABC$, being given, to draw a pa-

rallel, $DE$, to one of the sides $AC$, so that of the part $DE$ intercepted between the sides shall




have a given magnitude. Determine analytically the necessary construction, and shew from the formula that there are always two parallels which solve the problem.

10. To describe a parallelogram having a given angle, and whose sides shall be in a given ratio, and so that the vertices of two adjacent angles being on the base of a given triangle, the vertices of the remaining angles shall be on the remaining sides of the triangle. Shew analytically that *in general* two such parallelograms may be constructed, and determine the particular cases which admit but one.

11. Investigate the method of inscribing a square in a triangle, and determine the formula expressing the length of its side as a function of the base and altitude.

12. To determine the greatest square which can be inscribed in a given triangle.

13. To express the side of the inscribed square as a function of the three sides.

14. The parallelogram formed by tangents through the vertices of a system of conjugate

diameters in an ellipse or hyperbola, is equal to the rectangle under the axes.

15. Prove from this principle that the most oblique conjugate diameters are those whose rectangle is a maximum, and that the rectangle under the axes is less than the rectangle under any other system of conjugate diameters.

16. The equation $a^2 y^2 + b^2 x^2 = a^2 b^2$, expressing an ellipse or hyperbola, according as $b^2$ is positive or negative, related to its axes as axes of co-ordinates, to prove by the transformation of co-ordinates that there exists an indefinite number of systems of conjugate diameters, and to determine the equation of condition by which their mutual position is restricted.

17. To determine in general the condition under which the general equation of the second degree shall represent one or more right lines.

18. To determine in general the condition under which the general equation of the second degree becomes impossible.

19. To determine the conditions under which the axes of co-ordinates are tangents to the curve represented by an equation of the second degree.

20. To determine the points where a line of the second degree, given by its equation, intersects the axes of co-ordinates.

## PHYSICS.

21. If several concentric circles be described

round a centre where the force varies as the $n^{th}$ power of the distance, to determine the proportions of the periods and velocities.

22. What must be the law of the variation of the force, in order that the periods in all the circles may be equal?

23. What must be the law of the variation of the force, in order that the velocities may be equal?

24. Determine the law of the variation of the force, the periods being supposed proportional to the velocities.

25. Determine the law of the variation of the period, if the force be supposed proportional to the $m^{th}$ power of the velocity.

26. Determine the law of the variation of the velocity, if the force be supposed proportional to the $m^{th}$ power of the period.

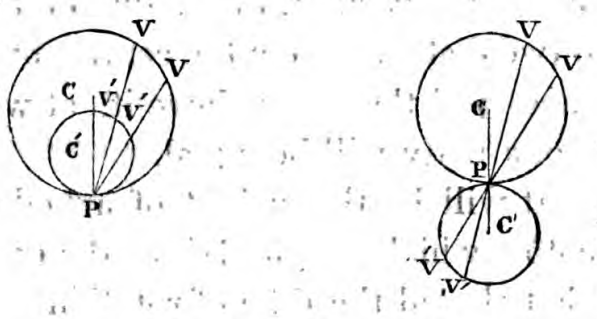
27. Given the intensity of the force of terrestrial gravity, to find the velocity of projection which would make a body projected horizontally become a satellite moving in a circular orbit.

## SECOND MORNING.

### PHYSICS.

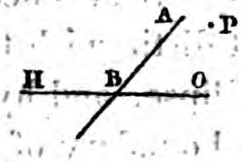
1. Two circles whose planes are vertical touch each other, so that the line joining their centres,

$CC'$, is vertical; if several chords, $PVV'$, be drawn through the point of contact, $P$, the parts




of these chords, $VV'$, intercepted between the circles will be described by falling bodies in the same time.

2. A right line, $AB$, is given in position in a vertical plane, to determine the right line of




quickest descent from a given point, $P$, in that plane to the given line $AB$.

3. A circle is given upon a vertical plane, to determine the right line of quickest descent from a given point in the plane to the circle.

4. Two circles are given in magnitude and position, on the same vertical plane, to determine the right line of quickest descent connecting points in their circumferences.

5. A circle and right line are given upon a vertical plane, to determine the line of quickest descent connecting them.

6. To find the altitude above the surface of the earth at which a body will lose half its weight.

7. To find the relation the radius of the earth bears to two distances from the centre, one within the surface, and the other above it, at which the weights of a body will be equal.

8. What will be the weight of a body at the latitude 45°, which weighs 1*lb*. at the equator?

9. What should the time of rotation be, in order that bodies at the equator should lose their entire weights?

10. What is the image of a circular arc concentric with a spherical reflector or a lens?

11. Prove analytically that the image of a straight line in a spherical reflector or a lens is a conic section, and distinguish the cases in which it is a parabola, ellipse, and hyperbola.

12. Why is the boundary of the circle of light and darkness on the moon more uneven than that of the circle of vision?

13. What is the cause of the moon's libration in longitude, and how is its greatest quantity measured?

14. Why is the libration in longitude, at the empty focus of the moon's orbit, much more inconsiderable than at the earth.

15. Prove the equable description of areas of the earth round the sun, by comparing the variation of the sun's apparent diameter with his change of longitude.

16. Why is not the right ascension of a star found by computing the right ascension of the sun from his declination and the obliquity of the ecliptic, and observing the difference between his right ascension and that of the fixed star; and in what is the method of determining the right ascension by two observations of the difference of right ascension, given by Brinkley, superior to this method?

17. Determine the latitude of the place by observing two altitudes of the sun, and the time between them.

## SECOND EVENING.

### PHYSICS.

1. A material point being moved round a fixed centre of force, the radius vector connecting it with the centre, describes areas proportional to the times of its motion.

2. Prove that the centre, round which the areas are equably described, must be the centre of force.

3. Prove that in every orbit described round a fixed centre of force, the value of the force at every point is expressed by the square of the velocity divided by half the chord of the circle of

curvature which passes through the centre of force.

4. Prove that in a circular orbit, the centre of force being on the circumference, the force varies inversely as the fifth power of the distance.

5. Investigate the ratio of the velocity in such a circle at any distance, to the velocity in a circle with that distance as radius, and having the centre of force as its centre.

6. If a material point be moved in a circle by the influence of a force which acts in a system of parallels, to determine the law of the variation of the force, the velocity, and the period.

7. If the planets moved in circular orbits, having the sun as a common centre, and with periods, the squares of which are supposed proportional to the third powers of the radii, prove that the force directed towards the sun must vary inversely as the square of the distance.

8. In the same case, and from the same data, prove that the velocities would be inversely as the square roots of the distances.

9. A material point describes a logarithmic spiral by the influence of a centripetal force directed towards the pole of the spiral, to determine the law of the variation of the force.

10. Shew that in the same cases the velocity increases in the same ratio as the distance decreases.

11. What is the proportion of areas described in the same time in different ellipses having a common focus, and that focus the centre of force?

12. Investigate the law of force necessary to make a material point move in a parabola, the direction of the force being that of the diameters.

13. A material point being moved in a circle by the influence of a force directed to a point outside the circle, to investigate the points of least velocity, and the velocities at the points of contact of tangents through the centre of force.

14. A material point being moved in a circle by a force directed to a point within it different from the centre, to determine the points at which the efficacy of the force is least and greatest.

# OCTOBER.

## FIRST MORNING.

### GEOMETRY.

1. Given the equation of a right line,

$$y = ax + b,$$

to determine analytically the equation of a right line, perpendicular to it, passing through a given point.

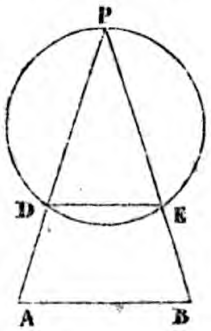
2. To investigate analytically a formula for the length of a perpendicular to a right line given by its equation.

3. To express the area of a polygon as a function of the co-ordinates of its several angular points, related to two rectangular axes of co-ordinates.

4. Investigate analytical formulæ for the radii of the circles inscribed in, and circumscribed about a given triangle, as functions of the sides.

5. Why has the formula expressing the radius of the inscribed circle four values?

6. To determine a point $P$, in a given circle,



*DPE*, such that two right lines, *AP*, *BP*, drawn from it to the extremities of a given right line, *AB*, the chord *DE* of the intercepted arc shall be parallel to the given right line *AB*.

7. By investigating the coordinates of the intersection of two right lines, determine the relation between the constants of their equations, which will render them parallel.

8. Determine the equation of a right line, inclined to a right line whose equation is given, at a given angle; and deduce from this the conditions under which the two right lines will be parallel or perpendicular.

9. The co-ordinates of the three vertices of a triangle related to two rectangular axes being given, to determine the co-ordinates of the point where the perpendiculars from the vertices on the sides intersect, and to shew that they have a common point of intersection.

10. By the same data to determine the co-ordinates of the point of intersection of the lines drawn from the vertices bisecting the sides, and to shew that they have the same point of intersection.

11. By the same data to determine the point of intersection of perpendiculars to the sides through these middle points, and to shew that they have the same point of intersection.

12. Prove that the co-ordinates of any one of these three points satisfy the equation of a right

line drawn through the other two, and therefore that the three points are *in directum*.

13. To determine the equation of a right line passing through a given point, such that the part of it intercepted between the axes of rectangular co-ordinates shall be a given length.

14. By the polar equation of a circle related to any origin within or without the circle, to shew that the rectangles under the segments of all chords through the same point are equal.

15. To express the area of a quadrilateral inscribed in a circle, as a function of its sides.

16. To draw a tangent to an ellipse, from a point outside it.

17. If $a$ and $b$ be the semiaxes of an ellipse, or hyperbola, the product of the tangents of the angles at which any system of conjugate diameters are inclined to the axis $2a$, measured in the same direction is $\mp \frac{b^2}{a^2}$, the sign $-$ applying to the ellipse, and $+$ to the hyperbola.

18. Prove from this that an hyperbola can have but two axes, and that if an ellipse has more than two axes, all diameters are axes, and the ellipse becomes a circle.

19. A tangent being drawn to an ellipse, and a perpendicular to it being drawn through the centre, to investigate the position of the tangent when the intercept between the perpendicular and the point of contact is a maximum.

20. A perpendicular to the tangent being drawn through the focus, to determine the position of the tangent, when the intercept between the point of contact and the perpendicular is a maximum.

21. If from the extremities of one system of conjugate diameters ordinates to another system be drawn, the parallelograms contained by these ordinates and the intercepts of the second system between the ordinates and the centre, are equal.

22. If a system of conjugate diameters be produced to meet a tangent, whose position is given, the rectangle under the intercepts of the tangent, between the conjugate diameters and the point of contact, is independent of the position of the conjugate diameters.

23. If a tangent be drawn to an ellipse, intersecting two parallel tangents and the produced diameter passing through their points of contact, the intercept of this tangent between the produced diameter and the point of contact, is an harmonical mean between the intercepts of the same tangent terminated by the produced diameter and the parallel tangents.

## FIRST AFTERNOON.

### TRIGONOMETRY.

1. Given three arcs, to express the sine and co-

sine of their sum, as functions of the sines and cosines of the arcs, and to express the tangent of the sum as a function of the tangents of the arcs.

2. To express sin. $A$ cos. $B$, and sin. $B$ cos. $A$, $A$ being supposed greater than $B$, as functions of the sum and difference of $A$ and $B$.

3. To express the tangent of half the sum of two arcs as a function of their sines and cosines.

4. To determine the arc, the product of whose sine and cosine is a maximum.

5. To determine the arc, the sum of whose sine and cosine is a maximum.

6. Prove that the square root of the cosine of an arc is a mean proportional between the sum and difference of the sine and cosine of half the arc.

7. Given the sum of two arcs, to determine when the sum of their sines is a maximum.

8. Given the sum of two arcs, to determine when the sum of their cosines is a maximum.

9. The product of the sum and difference of the sines of two angles is equal to the product of the sines of the sum and difference of the angles.

10. The quote of the sine of the sum and sine of the difference of two angles is equal to the quote of the sum of their tangent, and the difference of their tangents.

11. To express the sine of an angle as a function of its tangent.

12. To express the secant of an angle as a function of its sine.

13. To determine the value of the angle, the sum of whose tangent and co-tangent is a minimum.

14. To determine the angle, the sum of the squares of whose secant and cosine is a minimum.

15. Given two sides and the included angle of a plane triangle, to determine the base by formulæ adapted for the application of logarithms.

16. What are the peculiar advantages of the different methods of computing an angle of a plane triangle, the three sides being given.

17. Prove Neper's rules.

18. Explain and prove Neper's analogies.

19. What are the limits of the value of the sum of the angles of a spherical triangle?

20. What is the limit of the sum of the three sides of a spherical triangle?

21. Express the cosine of the angle of a spherical triangle, as a function of the sines and cosines of the sides.

## SECOND MORNING.

### PHYSICS.

1. Prove that the time of oscillation of a cycloidal pendulum is independent of the length of

the arc of vibration, and express this time as a function of the length of the string.

2. How does the pendulum prove the rotation of the earth?

3. What is the limit of the ratio of the time of descending through a circular arc terminating at the extremity of the vertical diameter, to the time of descent through its chord?

4. Why is a circular pendulum tautochronous when the arcs of vibration are very small?

5. In a system of pulleys, each of which acts by a separate string, attached to a fixed point, find the number of pulleys necessary to raise one hundred weight by a power of a pound.

6. In a system of pulleys fixed in the same moveable block, and all acting by the same string, to find the number of pulleys necessary to raise a weight represented by 20, by a power represented by unity.

7. A double convex lens is placed at a given distance from a screen, to determine the position of the object, in order that its image may fall upon the screen.

8. What is the limit of the distance between the lens and screen, which renders the solution of the preceding question possible?

9. An object is placed at a given distance from a screen, to determine the place of a given convex lens between them, in order that the image shall fall upon the screen.

10. To determine the limit of the focal length of the lens which will render the solution of the preceding question possible.

11. Given the altitude of the highest point of the violet ray in the rainbow, to determine the altitude of the sun.

12. Why are the colours inverted in the second rainbow?

13. Prove that the altitude of a fixed star is greatest when on the meridian.

14. Why is not the longest day the day of greatest heat?

15. Admitting the reasoning of Brinkley, why does it not prove that the autumnal equinox is the hottest day?

16. Why is Boscovich's method of finding the refraction practically useless?

17. The degree of ellipticity of the apparent discs of the sun and moon being measured by the ratio of the vertical to the horizontal diameter, it encreases with the zenith distance of the centre of the disc; what function of the zenith distance expresses the variation of this ellipticity?

18. What are the equations which express the relation between the true, the eccentric, and the mean anomaly?

19. Prove that the equation of time vanishes, once between the place of aphelion and the autumnal equinoctial point, once between the winter solstitial point and the place of perihelion,

once between the first of Aries and the point of the ecliptic whose longitude is such that the square of its tangent equals the secant of the obliquity, and once more between the latter point and the summer solstitial point.

20. Enumerate the arguments for the rotation of the earth on its axis in $23^h\ 56'$.

## SECOND EVENING.

### PHYSICS.

1. Express the ratio of the areas described in the same time by the planets, as a function of the transverse axis and eccentricity, and shew that in orbits of small eccentricity this ratio is nearly that of the square roots of the transverse axes.

2. The velocity of a point moving in a logarithmic spiral, the pole of the curve being the centre of force, is always equal to the velocity in a circle at the same distance.

3. What is the proportion of the areas described in the same time, in different logarithmic spirals, round the same centre of force?

4. In a logarithmic spiral, determine the time in which the radius vector passing through the moving point describes 360°, and shew that the times of every successive revolution of the radius

vector are in geometrical progression, and find the common ratio.

5. Determine from these principles the time of arriving from a given position at the pole of the spiral, by the summation of a decreasing geometrical series, the number of whose terms is infinite.

6. Determine the height from which a body must fall to a logarithmic spiral, in order to acquire when it meets the curve the velocity it would have at that point if it moved in the curve.

7. If a part be taken on the radius vector of a logarithmic spiral measured from the centre, always proportional to the velocity of a body moving in the spiral at the extremity of the radius vector, to determine the curve on which the extremity of the assumed part will be placed.

8. Extend the Eighth Proposition of Newton's Principia to the case where the force acts in the direction of the ordinates to a diameter of any conic section.

9. Determine the variation of the velocity in this case as a function of the ordinate.

10. Investigate the law of force acting parallel to the asymptote of an hyperbola, by which a body would be moved in that curve.

11. Determine the velocity in this case as a function of the ordinate.

12. To determine the height from which a body must fall externally towards an ellipse, the

centre of force being at the focus, in order when it meets the curve to have the velocity it would have if it were moving in the curve.

13. If a body be moved in an hyperbola by a repulsive force which decreases as the square of the distance increases, what will be its velocity at an infinite distance?

14. In the same case, if the body be projected from the curve towards the centre of force with the velocity in the curve, at what distance will it begin to return to the curve; and shew that the extremity of this distance is in the arc of a circle, of which the focus is the centre, the transverse axis the radius, and the angle it subtends at the centre equal the angle under the asymptotes.

15. In the same case, if a body be placed upon the curve, and be moved directly from the centre by the influence of the central force, its velocity will be always increasing, but never exceed a certain limit.

16. If a body be moved in an equilateral hyperbola by a repulsive force directed from the centre, to shew that the force and velocity vary according to the same law.

17. Shew that in concentric equilateral hyperbolas the areas described in the same time are as the squares of the transverse axes.

18. If a projectile moving in its parabolic path be projected from any point vertically upwards

with the velocity it has at that point, to determine the height to which it will rise.

19. To determine the horizontal range of a projectile, being given the angle of elevation and velocity of projection.

# 1817.

## APRIL.

### FIRST MORNING.

#### TRIGONOMETRY.

1. If two right-angled spherical triangles have a common acute angle, the cosines of the hypothenuses are inversely as the tangents of the angles not common.

2. Solve the equation

$$x^3 + p\,x + q = 0$$

by the angle, the tangent of which is $\frac{2\,p^{\frac{3}{2}}}{3^{\frac{3}{2}}\,q}$.

3. If $a$, $b$, $c$, be the sides of a spherical triangle, and $A$, $B$, $C$, the opposite angles,

tan. $a$ (sin. $C$ cot. $A$ + cos. $C$ cos. $b$) = sin. $b$.

4. In a spherical triangle given the base, the sum of the sides, and the difference of the base angles, to determine the triangle.

5. In a spherical triangle given the base and sum of the sides, what are the limits of the magnitude of the vertical angle?

6. If two right-angled spherical triangles have a common side, the sines of the sides not common are inversely as the tangents of the adjacent angles.

7. Given a side and the adjacent angle of a

spherical triangle, to express the ratio of the co-temporaneous increments of the remaining angles.

8. Given the two base angles of a spherical triangle, to find when a given error in observing the vertical angle shall produce the least possible error in computing one of the sides.

9. Given the hypothenuse of a right-angled spherical triangle to construct it, so that the sum of the tangents of the base angles shall be a minimum.

10. Prove that the sum of the squares of the cosines of the angles which the diagonal of a rectangular parallelopiped makes with the sides is equal to unity.

11. If two spherical right-angled triangles have the same hypothenuse, the cosines of their acute angles are as the tangents of the containing sides.

12. In the same case, the cosines of the sides are reciprocally proportional.

13. If $\pi$ represent a semi-circle, determine the sine, cosine, &c. of $\frac{4n+3}{2}\pi$; $n$ being supposed a positive integer.

14. Give a general formula, including all the arcs whose sines are positive.

15. Prove the formulæ

$$\sin.(n+1)a+\sin.(n-1)a=2\sin.na\cos.a.$$

## FIRST EVENING.

### TRIGONOMETRY.

1. Prove the series

$$x = \frac{\text{tan. } x}{1} - \frac{\text{tan. } {}^3x}{3} + \frac{\text{tan. } {}^5x}{5} \ldots\ldots$$

2. Determine the sum of the series

$$\cos. a + \tfrac{1}{2} \cos. 2a + \tfrac{1}{3} \cos. 3a + \tfrac{1}{4} \cos. 4a \ldots\ldots$$

3. If $a$, $b$, $c$, be the sides, and $A$, $B$, $C$, the angles of a spherical triangle, and $s = \frac{a+b+c}{2}$, the radius being unity.

$$\text{Tan. } \tfrac{1}{4}(A + B + C - 180^\circ) = \sqrt{\text{tan. } \frac{s}{2} \text{ tan. } \frac{s-a}{2} \text{ tan. } \frac{s-b}{2} \text{ tan. } \frac{s-c}{2}}.$$

4. If two plane triangles have two sides in the one equal to two sides in the other, and the included angles in the ratio of two to one, in what will be the ratio of the opposite sides?

5. If two spherical right-angled triangles have a common side, the sines of the angles adjacent to the common side are proportional to the cosines of the angles opposite to it.

6. If the vertical angle and the sum of the sides of a spherical triangle are given, the tangent of half the difference of the base angles varies as the sine of half the difference of the sides.

7. If the difference of the sides of a spherical

triangle and the included angle be given, the tangent of half the sum of the base angles varies inversely as the cosine of half the sum of the sides.

8. If the sum of the sides of a spherical triangle and the included angle be given, the cosine of half the difference of the base angles bears a given ratio to the cosine of half their sum.

9. Shew by this principle that if the sum of the sides and the included angle be given, the sum of the base angles is a minimum when the triangle is isosceles.

10. If the base and the sum of the base angles of a spherical triangle be given, the sine of the vertical angle bears a constant ratio to the sum of the cosines of the sides.

11. If a given angle be divided into any two parts, the sum of the sines of those parts bears a constant ratio to the sum of their cosines.

12. Three points being given upon a plane to determine a fourth, by the angles which lines, from the sought point to the given points, make with each other.

13. Express the area of a plane triangle as a function of two of its angles and one of its sides.

14. The area of a regular polygon inscribed in a circle being given, and the area of one circumscribed with the same number of sides, to determine the area of polygons inscribed and circumscribed having double the number of sides.

15. To express the area of a triangle as a

function of two sides, and the angle opposite one of them.

16. The side of a triangle inscribed in a circle is to the radius of the circle as double the sine of the opposite angle is to unity (radius).

## GEOMETRY.

1. The co-efficient of the product of the variables, in an equation of the second degree, being wanting, to determine the position of the axes of co-ordinates with respect to the curve.

2. What does the general equation of the second degree represent when the first three terms form a complete square?

3. In an equation of the second degree, in which the square of $y$ is wanting, what is the curve and the position of the axes of co-ordinates with respect to it?

4. In an hyperbola if two right lines be drawn cutting it, one parallel to an asymptote, and the other parallel to a line given in position, a fourth proportional to the segment of the first between the intersection of both and the curve, and the two segments of the other shall be a line of an invariable length.

5. What is the limit of the length of the normal in the parabola?

6. Determine the curve, from every point of which perpendiculars drawn to a given parabola are equal.

7. The ordinates to the axis of a conic section are cut in a given ratio, to determine the locus of the point of section.

8. The projection of a circle upon a plane is an ellipse.

9. Explain the properties of supplemental chords, and shew their application to solve the following problems:

1°. To draw the conjugate diameters at a given angle.

2°. To draw a diameter conjugate to a given one.

3°. To draw a tangent to a given point on the curve.

## SECOND MORNING.

1. Find the equation of the evolute of the parabola.

2. Determine the point of the greatest curvature in the logarithmic.

3. Find the area intercepted between two ordinates to the asymptote of the logarithmic.

4. Shew that the curvature of the logarithmic spiral varies inversely, as the radius vector from the pole.

5. Determine the involute and evolute of the logarithmic spiral.

6. To draw a tangent to the spiral of Archimedes, by geometrical construction.

7. Determine the polar sub-tangent in the spiral of Archimedes.

8. Shew that the obliquity of the tangent to the radius vector in this spiral continually diminishes as the describing point recedes from the pole.

9. To determine the locus of a point, from which tangents being drawn to a given parabola, their sum shall be invariable.

10. Given the focus of a parabola, and a point through which it passes, to determine the locus of the vertex.

11. Given in position the centre of an hyperbola, and a point through which it passes, and in magnitude its transverse axis, to determine the locus of the foci.

12. Determine the locus of a point from which tangents drawn to a given parabola shall be in a constant ratio.

13. Given the base of a triangle, the sum of the sides, and the difference of the angles at the base, to construct the triangle by the intersection of two loci.

14. Determine the position of the centre and axes of an hyperbola represented by the equation,

$$Ay^2 - Cx^2 + Dy + Ex + F = 0.$$

15. Shew that this equation cannot represent an hyperbola unless $A$ and $C$ have different signs.

16. Shew that the directions of the asymptotes is independent of the values of $D$, $E$, and $F$.

17. Determine the effect produced upon the position of the centre, by the supposition that $D$ and $E$ are negative.

18. Determine the effect produced upon the position of the centre, by the supposition that $D$ and $E=0$.

19. Determine the position of the origin of co-ordinates with respect to the curve when $F=0$.

20. Shew that the axes of this hyperbola are parallel to the axes of co-ordinates.

21. In a series of lines in decreasing geometrical progression, continued *ad infinitum*, given the sum of the series and the second term to find the other terms, and shew in what respect the solution is equivocal.

22. In a like series, given the sum of the series, and the difference of the first two terms to determine the terms severally.

23. In a like series, given the difference of the first two terms, and the ratio, to determine the sum.

24. In a like series, given the sum of the series and a geometrical mean between the sum and the second term, to find the terms severally.

25. A series of this kind being given, to determine the sum of the squares of all the terms, and determine a line whose square is equal to it.

26. Find a general formula for the perpendicular from the origin of co-ordinates on the tan-

gent through any point, in terms of the co-ordinates of that point, and their differentials.

27. Find a similar formula related to polar co-ordinates.

28. Given the base and vertical angle of a triangle, the line joining the vertex with the point of intersection of the perpendiculars on the sides from the opposite angles, is of an invariable length.

29. The locus of the extremity of the polar subtangent is a straight line; determine the curve.

30. Determine the spiral of which the area is the logarithm of the radius vector.

31. Determine the asymptote of the lituus.

## SECOND EVENING.

### PHYSICS.

1. Two concave spherical reflectors are constructed with equal radii, what should be the proportion of their magnitudes in order that one should produce the same burning power with moon-light as the other does with sun-light, supposing the reflecting power of the moon's surface to be given?

2. A body is projected so that its horizontal range is a maximum, and another oscillates in a

cycloid, the axis of which is the height due to the velocity of projection, what is the ratio of the time of flight of the one body, to the time of vibration of the other?

3. Determine the eccentricity of the orbit of the earth from observations on the diameter of the sun.

4. Determine the variation of the moon's apparent diameter, at different altitudes, owing to the change of the spectator's distance from her?

5. What are the presumptions for and against the existence of an atmosphere on the moon?

6. Compute that part of the aberration of a planet which is due to the planet's own motion.

7. Shew that a total and central eclipse of the moon is shorter, the longer the moon's path in the shadow is.

8. Explain the method of finding the latitude at sea, when the sun has not been visible at noon.

9. Explain the lunar method of finding the longitude, and the method of reducing the observed angular distance to what it would be at the centre.

10. Shew that the force cannot vary inversely as the square of the distance, whether it be attractive or repulsive, except in a conic section, of which the focus is centre of force.

11. When is the illuminated part of the disc of a superior planet least?

12. Explain the different methods of correcting the line of collimation of a telescope.

13. Explain and prove the method of finding the longitude for geographical purposes, by observing the increase of the moon's right ascension in the interval of her transit over two meridians.

14. Find the length of a pendulum which would vibrate in the time of a satellite at the surface.

15. Prove analytically that the times of describing successive equal spaces by a body moved *in vacuo*, by an uniform force, are represented by the odd integers.

16. If two bodies move down two inclined planes, having a common altitude, and set off together from their intersection, the line joining the bodies moves parallel to itself.

17. Will this hold good if the planes be supposed to produce a friction proportional to the perpendicular pressure, the weights of the bodies and the intensity of the friction being supposed to be given?

18. Given the density of the air in the receiver of an air-pump, and also the magnitude of the receiver and barrel to determine the number of turns which have been made.

19. A plane moves in a fluid perpendicularly to its surface, how does the resistance of the fluid vary?

20. A weight rests upon the circumference of a

circle, and is sustained by another, by a string passing over a fixed pulley; the weights being given, determine the point of the circle at which they will be in equilibrium.

21. In the same system to determine the relation of the weights which will produce equilibrium at a given point upon the circle.

22. Determine the shape of the caustic by reflection, when the reflecting curve is a circle, and the focus at the middle point of a radius; and shew that it has three cusps and an asymptote.

23. When the focus of incident rays is at a distance from the centre greater than one-half of the radius, shew that the real caustic has three cusps and two asymptotes.

24. If the focus of incident rays be at a distance from the centre less than one-half of the radius, shew that the caustic will have four cusps and no asymptote.

25. What is the caustic of a reflecting cycloid, the rays being parallel to its axis?

26. If $A$ and $B$ be two elastic bodies, placed in the same straight line, to determine the velocity communicated to $B$, by $A$ moving with the velocity $a$, through an infinite number of mean proportionals between $A$ and $B$.

27. Two unequal weights act over a fixed pulley to determine the velocity which the heavier will acquire in any given time.

28. If a body be attracted towards two centres of force, and projected in any direction not coinciding with a plane passing through the line which joins the centres of force, shew the relation between the times of its motion and the solid described by the triangle, whose base is the line joining the centres of force, and whose sides are the lines joining the place of the body with those centres.

29. To determine the specific gravity of air by the air-pump.

30. Determine in general the point of an orbit at which the velocity equals the velocity in a circle at the same distance.

31. Shew that at this point the condition expressed by the equation

$$dz^2 = z d^2 z$$

must be fulfilled, $z$ being the radius vector.

32. Shew that at this point the angular velocity of the radius vector is equal to the angular velocity of the perpendicular upon the tangent.

33. Determine the law of the force, directed to the pole, in a curve, the polar equation of which is

$$z^2 = a^2 \cos. 2\omega,$$

# OCTOBER.

## FIRST MORNING.

### TRIGONOMETRY.

1. GIVEN the logarithms of the three sides of a plane triangle, to determine the logarithms of the segments of one of them, made by the bisector of the opposite angle.

2. In the series

$$\text{sin. } m\,a = m \text{ sin. } a - \frac{m \,.\, m^2 - 1}{1 \,.\, 2 \,.\, 3} \text{ sin. } {}^3a + \frac{m \,.\, m^2 - 1 \,.\, m^2 - 9}{1 \,.\, 2 \quad . \quad 3} \text{ sin. } {}^5a \ldots\ldots$$

to determine the last term.

3. Determine the value of $x$ in the equation,

$$\text{sin. } (x + a) + \text{cos. } (x + a) = \text{sin. } (x - a) + \text{cos. } (x - a).$$

4. Determine the value of the series

$$1 + m + \frac{m \,.\, m - 1}{1 \,.\, 2} + \frac{m \,.\, m - 1 \,.\, m - 2}{1 \,.\, 2 \quad . \quad 3} \ldots\ldots$$

5. Given the radius of a circle inscribed in a plane right angled triangle of a given area, to determine the sides of the triangle.

6. Determine the value of $x$ in

$$\text{sin. } (30° + x) + \text{sin. } (30° - x) = \tfrac{1}{2}.$$

7. In the series

$$2^{m-1}\cos.{}^m a = \cos. ma + m\cos.(m-2)a + \frac{m\,.\,m-1}{1\,.\,2}\cos.(m-4)a \ldots\ldots,$$

to determine the last term.

8. In a plane triangle two sides being given, to find when a given error in the observation of the included angle will produce the least possible error in computing the opposite side.

9. In a plane triangle two sides being given, to determine the ratio of a small variation in the included angle to the corresponding variation of one of the base angles.

10. To determine when a given error in the observation of an angle produces the least possible error in the computation of its secant.

11. If two right angled spherical triangles have a common acute angle, the tangents of the sides which include it are in a given ratio.

12. Solve the equation

$$x^2 - px = -q,$$

by the angle whose sine is $\frac{2\sqrt{q}}{p}$.

13. Solve the equation

$$x^3 - px - q = 0$$

by the angle, the sine of which is equal to $\frac{2p^{\frac{3}{2}}}{3^{\frac{3}{2}}q}$.

14. If $a$, $b$, $c$ be the sides and $A$, $B$, $C$ the angles of a spherical triangle

$$\frac{\sin. B}{\tan. C} = \sin. a \cot. c - \cos. a \cos. B.$$

## FIRST EVENING.

### TRIGONOMETRY.

1. If $a$, $b$, $c$ be the sides, and $A$, $B$, $C$ the angles of a spherical triangle

$$\sin. c = \tan. b \,(\sin. A \cot. B + \cos. A \cos. c.)$$

2. In a spherical triangle given the base, vertical angle, and the difference of the base angles, to determine the triangle.

3. Given an angle of a spherical triangle, and one of the sides containing it, what are the limits of its area?

4. Determine the sum of

$$\sin. (a + d) + \sin. (a + 2d) + \sin. (a + 3d) \cdots$$

continued *ad infinitum*.

5. Determine the sum of

$$\cos. (a + d) + \cos. (a + 2d) + \cos. (a + 3d) \cdots$$

continued in *infinitum*.

6. Prove the formula

$$\cos.^2 a - \tfrac{1}{2} = \cos. (45° + a) \cos. (45° - a.)$$

7. If $1 + \sin. a = 2 \cos.^2 (x - \frac{1}{2} a)$

what is $x$?

8. Prove the series

$$\pi = 4 \{1 - \tfrac{1}{3} + \tfrac{1}{5} - \tfrac{1}{7} \cdots\cdots\}$$

and find from it a series sufficiently convergent for computation.

9. If two right angled spherical triangles have a common side, the tangents of the angles adja-

cent to the common side are as the tangents of the opposite sides.

10. Given a side and the adjacent angle in a spherical triangle, to determine the ratio of the increment of the side containing the given angle, to the simultaneous increment of the angle opposite the given side.

11. Given the two base angles of a spherical triangle to find when a given error in observing one of the opposite sides, shall produce the least possible error in computing the other.

12. To find a point within a given spherical triangle, which is equidistant from its three angles, and to express its common distance from the angles as a function of the sides.

13. To deduce from this by supposing the radius of the sphere infinite, the expression for the radius of a circle circumscribing a plane triangle.

14. To find a point within a spherical triangle, from which secondaries to the three sides being drawn, the arcs of these between the assumed point and the sides will be equal, and to express these arcs as functions of the sides.

15. To deduce from this the expression for the radius of a circle inscribed in a plane triangle, as a function of the sides, by supposing the radius of the sphere infinite.

16. Find the sum of the series

$$\cos. a + \cos. 3a + \cos. 5a \cdots\cdots$$

continued to $n$ terms.

17. Find the sum of the series

$$\cos. a + \cos. (a+h) + \cos. (a+2h) \cdots\cdots$$

continued to $n$ terms.

18. Find the sum of the series

$$\sin. 2a + \sin. 4a + \sin. 6a \cdots\cdots$$

continued to $n$ terms.

19. Find the sum of the series

$$\sin. a + \sin. 3a + \sin. 5a \cdots\cdots$$

continued *ad infinitum*.

## SECOND MORNING.

### GEOMETRY.

1. To determine the position of the axes of co-ordinates with respect to a line of the second degree, when the co-efficients of $y$ and $x$ in the equation respectively equal 0.

2. If the co-efficients of the squares of the variables have different signs in an equation of the second degree, what is the species of the locus?

3. If two tangents to a conic section be drawn from the same point, and also a right line joining the points of contact, the intercept of any parallel to one tangent between the other and the line joining the points of contact, is a mean proportional between the intercepts of the same line between the tangents and the curve.

4. If several chords be drawn through the focus of a conic section, and through the points where

these intersect the curve tangents, be drawn until they meet, what right line is the locus of their intersection?

5. Prove that there are no foci on the conjugate axis of an hyperbola.

6. What are the limits of the length of the normal in the ellipse?

7. Determine the curve from every point of which perpendiculars drawn to a given ellipse are equal.

8. Find the equal conjugate diameters by the properties of supplemental chords.

9. In an ellipse, or hyperbola, what is the invariable ratio of the normal to the conjugate diameter?

10. Determine the ratio of the normal of the cycloid to the radius of curvature.

11. To trisect an angle by means of the cycloid.

12. What are the values of the radius of curvature at the extremities of the axes of an ellipse?

13. Shew that these are the points of greatest and least curvature.

14. Determine the point of greatest curvature in the cycloid.

15. Find the equation of the evolute of the ellipse.

16. Explain the method of finding two mean proportionals by the Cissoid of Diocles.

17. To draw a tangent to the Conchoid by geometrical construction.

18. Determine the area included between two radii vectores of the spiral of Archimedes.

19. Determine the spiral in which the polar sub-tangent is constant.

20. Given one focus of an ellipse, a point through which it passes, and the position of the tangent at that point, to determine the loci of its vertices.

21. Given the position of the vertex of an hyperbola, a point through which the curve passes, and the position of a tangent at that point, to determine the locus of the foci.

22. Given, in position, the centre of an ellipse, in magnitude its greater axis, and a point through which it passes, to determine the locus of the vertices of the conjugate axis.

23. Given the base of a triangle, the difference of the sides, and the rectangle under them, to construct the triangle, by the intersection of two loci.

24. Given the base of a triangle, and the ratio of the rectangle under the sides to the difference of their squares, to determine the locus of the vertex.

25. Given the difference of the squares of two lines, and the rectangle under them, to determine them by the intersection of two loci.

26. Determine the distance between the centres of the circle inscribed in a triangle, and that

circumscribed round it, as a function of their radii.

27. A line revolves in a given plane round a fixed point, and the part intercepted between the sides of a given angle in the same plane is divided in a given ratio, to find the locus of the point of division.

28. To determine the locus of a point from which two tangents being drawn to a given ellipse their sum shall be invariable.

29. Determine the magnitude of a spherical surface intercepted between two parallel lesser circles.

30. Determine the solid contents of the segment of a sphere cut off by a lesser circle.

31. Determine the surface of a truncated cone.

32. Determine the ratio of the volumes of a pyramid and prism of equal bases and altitudes.

33. The segment of a circle being supposed to revolve upon its chord, to determine the volume of the solid of revolution thus produced.

34. Two segments of circles described upon the same chord, being supposed to revolve upon the line passing through their centres, to determine the volume of the solid of revolution so produced.

35. If any polygon be circumscribed about an ellipse, the continued products of the alternate segments of the sides made by the points of contact shall be equal.

36. If two polygons be circumscribed about the same ellipse, and their sides be respectively divided, each to each proportionally at the several points of contact, the areas of the polygons will be equal.

37. The inscribed polygons formed by the junction of the points of contact of the sides of these polygons with the ellipse, will be also equal.

38. Determine the least polygon of a given number of sides which can be circumscribed round an ellipse.

## SECOND EVENING.

### PHYSICS.

1. Two weights are connected by a string which plays across a fixed pulley, and are supported, one upon a circle, and the other upon such a curve that they balance each other in every position whatever; to determine the nature of the second curve.

2. A ball of a given elasticity is to be projected from a given point, so that being reflected by a number of given planes perpendicular to that on which it moves, in a given order, it may afterwards strike a given point.

3. To determine the right line of quickest de-

scent between two circles whose common plane is vertical.

4. To determine the time in which a given power will draw a given weight up a given inclined plane, the effects of friction being included.

5. The weight and power being given to determine the elevation of the plane, in order that the power may act to the greatest possible advantage in the elevation of the weight.

6. If equal centripetal forces tend to the several points of a spherical surface, varying inversely as the squares of the distances, a particle placed within the surface is not attracted by any part of these forces.

7. A cycloid being supposed to be described by the generating circle rolling on the base, determine the ratio of the velocity of the generating point to the velocity of the centre of the generating circle.

8. Given the lengths of a division of a vernier, and a division of the instrument to which it is attached; find the least magnitude the vernier is capable of measuring.

9. Given the same quantities; of what length should the vernier be?

10. In reducing the transit instrument to the meridian, why is one half the apparent error corrected by moving the supports of the cross axis, and the other half by moving the wires?

11. Explain the effect of each of these motions on the position of the line of collimation.

12. Explain the method of finding the meridian by equal altitudes of a star of known right ascension, and the time between the two observations.

13. A pendulum falls from the horizontal position, to determine the point of its descent, at which the force stretching the rod is equal to the force which accelerates the bob.

14. Determine the force which stretches the rod at the lowest point in the same case.

15. What is the quantity and position of a force which is compounded of two forces which are equal, and act in opposite directions in two parallel straight lines?

16. What is the method of computing a planet's distance from the sun?

17. If a body be moved in an ellipse by a force parallel to one of the axes, what is the law of the force?

18. A sphere floats between two fluids, its specific gravity being a geometrical mean between the specific gravities of the two fluids, to find the ratio of the segments into which its vertical diameter is divided by the common surface of the fluids.

19. How is the length of the horizontal diameter of the sun affected by refraction?

20. Considering the effects of refraction, does

a star near the horizon move in a parallel of declination?

21. Is refraction instrumental in causing twilight, and how?

22. If two perfectly inelastic bodies, $A$ and $B$, move in the same straight line with the velocities $a$ and $b$, determine the velocity lost by one and that gained by the other after impact.

23. If an imperfectly elastic body impinge upon a perfectly hard plane, to determine the relation between the angles of incidence and reflection as a function of the elasticity.

24. To determine the number of strokes of the air pump which will produce a given degree of rarefaction in the air contained in the receiver.

25. Determine the ratio of the heat in the focus of a convex lens to the sun's direct heat.

26. If differently coloured solar rays had different velocities, what would be the effect produced on the aberration of the fixed stars?

27. Determine the law of the force directed to the pole of the spiral represented by the equation.

$$\frac{\sqrt{a^2+z^2}-a}{\sqrt{a^n+z^n}+a}=e^{\omega}$$

where $z$ represents the radius vector, and $\omega$ the angle which it makes with a fixed axis.

28. Determine in general an equation between the centripetal and centrifugal forces and the paracentric velocity.

29. Shew by means of this equation that the paracentric velocity is always a maximum when the centripetal and centrifugal forces are equal.

30. Determine in general the point in an orbit where the angle of projection is a minimum.

# 1818.

## APRIL.

### FIRST MORNING.

#### TRIGONOMETRY.

1. GIVEN the three sides of a plane triangle, to determine formulæ for the segments of one of them made by a perpendicular from the opposite angle, which shall be fit for the application of logarithms.

2. In the series

$$\text{sin. } ma = m \text{ sin. } a - \frac{m \cdot m^2-1}{1 \cdot 2 \cdot 3} \text{ sin. } {}^3a + \frac{m \cdot m^2-1 \cdot m^2-9}{1 \cdot 2 \cdot 3 \cdot 4 \cdot 5} \text{ sin. } {}^5a \cdots\cdots$$

to determine the number of terms.

3. Determine the value of $x$ in the equation.

$$\frac{\text{sin. } (x+a) + \text{sin. } (x-a)}{\text{cos. } (x+a) + \text{cos. } (x-a)} = \sqrt{3}.$$

4. In a plane right-angled triangle, given the radius of the inscribed circle and the perimeter to determine the triangle.

5. In a spherical right-angled triangle, given the hypothenuse and the difference of the sides to determine the triangle.

6. To determine when a given error in the ob-

servation of an angle produces the least possible error in the computation of the tangent.

7. If two right-angled spherical triangles have a common acute angle, the cosines of the angles not common are as the cosines of the opposite sides.

8. The difference of the tangents of two angles is to the difference of their co-tangents as the product of their sines is to the product of their cosines.

9. Solve the equation

$$x^2+px=-q$$

by the angle whose sine is $\frac{2\sqrt{q}}{p}$

10. Solve the equation

$$x^3-px+q=0$$

by the angle, the sine of thrice which, is $\frac{2p^{\frac{3}{2}}}{3^{\frac{3}{2}}q}$.

11. Solve the equation

$$x^3-px-q=0$$

by the same angle.

12. If $a$, $b$, $c$, be the sides, and $A$, $B$, $C$, the opposite angles of a spherical triangle,

$$\frac{\text{sin. } A.}{\text{tan. } C.}=\text{sin. } b \text{ cot. } c-\text{cos. } b \text{ cos. } A.$$

13. Also

$$\text{sin. } a=\text{tan. } c. \text{ (sin. } B \text{ cot. } C+\text{cos. } B \text{ cos. } a.)$$

14. In a spherical triangle, given the base vertical angle and the difference of the sides to determine the triangle.

## FIRST EVENING.

### TRIGONOMETRY.

1. Solve the equation

$$ax^4+4x^3-6ax^2-4x+a=0$$

by the angle whose tangent is $a$.

2. If $1-\sin. a=2\sin.^2(x-\frac{1}{2}a)$ what is $x$?

3. Find analytically the cosine of 60°.

4. Find analytically the sine of 45°.

5. Given the base and sum of the sides of a plane triangle, to determine when the vertical angle is a maximum.

6. Given the hypothenuse and difference of the sides of a right-angled spherical triangle, to determine the sides and angles.

7. If two spherical right-angled triangles have a common side, the sines of the hypothenuses are inversely as the sines of the angles opposite the common side.

8. In a spherical triangle, given a side and an adjacent angle to express the ratio of the variation of the side opposite the given angle, and the simultaneous variation of the angle opposite the given side.

9. Given the two base angles of a spherical triangle, to find when a given error in observing one of the sides shall produce the least possible error in computing the vertical angle.

10. The external angle of any spherical tri-

angle is less than the sine of the two internal remote angles.

11. In all quadrantal triangles, the angles are of the same affections as the opposite sides.

12. The angles and sides of a quadrantal triangle are respectively of the same affection, or different affections, as the hypothenuse is greater or less than a quadrant.

13. If one of the acute angles in a right-angled spherical triangle be given, the hypothenuse is greater than the containing side, equal to it, or less than it, according as the hypothenuse is less than a quadrant, equal to a quadrant, or greater than a quadrant.

14. An acute angle in a right-angled spherical triangle being given, to find when the opposite side is a maximum.

15. Given the logarithm of three numbers to find a fourth proportional to these numbers.

16. Given the logarithms of two numbers to find any number of geometrical means between them.

17. Explain the origin of the term *hyperbolic* logarithm.

18. What are *logistic* logarithms?

19. What determines the signs of the secant and co-secant of an arc?

20. What determines the signs of the tangent and co-tangent of an arc?

GEOMETRY.

21. Shew that the origin of co-ordinates cannot be at the centre of a line of the second degree, if the terms $Dy$ or $Ex$ occur in its equation.

22. Prove that every equation of the second degree which represents an ellipse must involve the squares of both variables with co-efficients which have like signs.

23. If the co-efficients of the squares of the variables be equal, and have the same sign, the axes of co-ordinates can be assumed at such an angle, that the curve represented by the equation must be a circle.

24. Find the limits of the rectangle under a system of conjugate diameters.

25. When is the subnormal in the hyperbola a minimum?

26. If a right line intersect an hyperbola and its asymptotes, the intercepts between the curve and asymptotes are equal.

27. A line being drawn from any point in an hyperbola, parallel to an asymptote, and equal to the distance from the focus, to determine the locus of its extremity.

28. Given in position the two sides of a triangle, and the angles in magnitude, to find the locus of the point which divides the base, so that

the rectangle under the segments shall have a given magnitude.

29. Given in position two sides of a triangle, the base of which passes through a given point, to find the locus of a point which divides the base, so that the sum of the squares of the segments shall have a given magnitude.

30. The rectangle under the segments of a chord passing through the focus, varies as the sum of the segments.

## SECOND MORNING.

1. Determine the radius of curvature at the vertex of a parabola, and shew that this is the point of greatest curvature.

2. Find the equation of the evolute of the hyperbola.

3. Apply the conchoid to the trisection of an angle.

4. To draw a tangent to the Lemniscate, by geometrical construction.

5. Shew that the area of this curve is equal to the square of its semi-axis.

6. Find the sum of all the sines in the semicircle.

7. Why is the logarithmic spiral called the equi-angular spiral?

8. Determine the length of an arc of this curve, continued from a given point to the pole.

9. Given one focus of an ellipse, a point through which it passes, and the magnitude of its greater axis, to determine the locus of its centre.

10. Given the centre of an hyperbola, a point through which it passes, and the position of a tangent at that point, to determine the locus of the foci.

11. Given the base of a triangle, the ratio of the sides, and the rectangle under them, to construct the triangle, by the intersection of two loci.

12. Given the base of a triangle, the difference of the base angles, and the difference of their sines, to construct the triangle, by the intersection of two loci.

13. Determine the parameter, focus, and the axis of the parabola represented by the equation

$$Ay^2 + Dy + Ex + F = 0$$

the co-efficients being all supposed positive.

14. How will the supposition that $A$ and $D$ have different signs, limit the position of this curve?

15. What relation has the axis of $x$ to this curve?

16. What modification would this curve undergo if $D=0$?

17. What effect would the supposition $F=0$ produce upon the position of this curve?

18. Two circles are given in magnitude and position, in the same plane, to determine the locus of a point, from which tangents being drawn to the circles, their sum is invariable.

19. To determine the locus of a point from which tangents drawn to a given parabola shall have a given difference.

20. To determine the locus of a point, from which two tangents being drawn to a given hyperbola, their sum shall be invariable.

21. Two tangents drawn from the same point to a given parabola, have a given ratio, to determine the locus of the point.

22. Given one focus of an ellipse, a point through which it passes, and its greater axis, to determine the locus of the other focus.

23. Given the centre of an hyperbola, a point through which it passes, and the position of a tangent at that point, to determine the locus of its vertices.

24. Given the base of a triangle, the ratio of the sides, and the difference of the opposite angles, to construct the triangle, by the intersection of two loci.

25. In a decreasing geometrical series of lines, continued *ad infinitum*, given the first two, to determine the sum of the series.

26. In a like series given the sum of the series, and the ratio, to determine the terms severally.

27. In a like series given the $n^{th}$ term, and the sum of the series, to determine the other terms.

28. Given the sum of such a series, and the difference between any proposed terms, to determine the other terms.

29. A series of this kind being given, to determine the sum of the differences of every pair of successive terms.

30. Given the geometric mean, and the sum of the arithmetic and harmonic means, to determine the extremes.

31. Determine the ratio of the surface of a sphere to the area of one of its great circles.

32. Determine the ratio of the solid contents of a sphere to the cube of its radius.

33. Determine the solid contents of a truncated cone.

34. If a pyramid and prism reciprocate their bases and altitudes, what is the ratio of their volumes?

35. The segment of a circle being supposed to revolve upon its chord, to determine the surface of revolution so generated.

36. Two segments of circles described upon the chord being supposed to revolve upon the line passing through their centres, to determine the surface of the lens so produced.

## SECOND EVENING.

### PHYSICS.

1. The motions of bodies enclosed in a given space are not changed by the rectilinear motion of that space.

2. What is meant by quantities tending to equality, *tempore finito*?

3. Determine by Newton's first lemma the ultimate ratio of the difference of the squares of two quantities to their difference.

4. Determine the ultimate ratio of the difference of the ordinates to the difference of the abscissas.

5. Determine the ultimate ratio of the difference of the ordinates to twice the sagitta.

6. Determine the ultimate ratio of the increment of the area to the corresponding increment of the abscissa.

7. If a body be moved in an ellipse, by a force acting along a system of parallel lines, what is the law of the force?

8. A globular vessel, filled with a fluid, discharges itself through an orifice at the lower extremity of the vertical diameter; find the time it will take to empty itself.

9. A sphere floats between two fluids, and its vertical diameter is divided by their common sur-

face in extreme and mean ratio, to determine the proportion of the three specific gravities.

10. A sphere floats between two fluids, its specific gravity being an arithmetical mean between the specific gravities of the fluids, to determine the segments into which the vertical diameter of the sphere is divided by the common surface of the fluids.

11. If a body move in a conic section, the centre of which is the centre of force, to determine the velocity with which the ordinate to the axis passing through the body is moved.

12. Find the time of describing any arc of an ellipse, the centre of which is the centre of force.

13. Prove that when the force is at the focus of a conic section, it varies inversely as the square of the distance.

14. Determine the value of the chord of the osculating circle, which passes through the focus.

15. Determine the velocity in an ellipse, whose minor axis is evanescent.

16. Prove that in ellipses with the foci as centres of force, the synchronous areas are as the square roots of their latera recta.

17. Given the focus, the velocity, and the angle of projection, to determine the orbit.

18. Given the variation of the force in a circle,

with the centre as centre of force, to determine the variation of the period.

19. If a body move in a parabola, the force acting in the direction of the ordinates to the axis, what is the law of the force?

20. If the force act in the direction of the ordinates to the asymptote of an hyperbola, what is its law?

21. If a body describe a semi-circle by the influence of a force perpendicular to its diameter, to determine the time.

22. If a body move in a conic section, the centre of force being at the centre, determine the variation of the velocity.

23. Express the centrifugal force as a function of the chord of curvature passing through the centre of force.

24. Express the same by a function of the radius of the osculating circle.

25. Make these formulas applicable to different orbits.

26. Determine the velocity at any point in an orbit.

27. In a circle, the centre of force not being at the centre, determine the variation of the velocity.

28. If the force vary inversely, as the fifth power of the distance, what is the velocity acquired in falling from an infinite altitude?

29. If a body moves in a semicircle, the force acting perpendicularly to the diameter, to determine the variation of the velocity.

30. Shew that in ellipses of small eccentricity the angular velocity is nearly uniform.

31. Shew that in all ellipses the angular velocity round the empty focus is less subject to variation than the angular velocity round that focus in which the force resides.

32. Given the law of the period in a circle, with the centre of force at the centre, to determine the variation of the velocity.

33. Determine the tangential velocity of a satellite revolving at the surface of the earth.

34. Determine the quantity by which gravity at the equator, is diminished by the centrifugal force.

35. Determine the same at any proposed latitude.

36. Prove that the angle under the chord and tangent has no limit in its diminution.

37. Prove that the limit of the circle described through the extremities of three equi-distant ordinates is the osculating circle.

38. In the tenth lemma Newton states that the space described is represented by the area of a curve, whose ordinate represents the velocity, and whose abscissa represents the time; prove this.

39. The limit of the subtense of the angle of contact varies as the square of the conterminous arc.

40. Prove that the force in any curve is represented by twice the sagitta of the arc described in one second.

41. Determine, in general, the variation of the angular velocity.

# OCTOBER.

## FIRST MORNING.

### TRIGONOMETRY.

1. FIND the value of $x$ in the equation,

$$\sin.(x+a)+\sin.(x-a)=\cos.(x+a)+\cos.(x-a.)$$

2. In the series

$$2\cos.ma=(2\cos.a)^m-m(2\cos.a)^{m-2}+\frac{m.m-3}{1.2}(2\cos.a)^{m-4}.....$$

determine the number of terms.

3. Given two sides, and the included angle of a plane triangle, find the segments into which the bisector of the third side divides the included angle.

4. In the series

$$\frac{\sin.ma}{\cos.a}=m\sin.a-\frac{m.m^2-4}{1.2.3}\sin.^3a+\frac{m\,m^2-4\,.\,m^2-16}{1.2.3.4.5}\sin.^5a.....$$

determine the number of terms.

5. Given the base and vertical angle of a plane triangle, to determine the ratio of the variation of either side to the simultaneous variation of the angle opposed to that side.

6. In a plane right angled triangle, given the radius of the inscribed circle and the hypothenuse to determine the triangle.

7. In a spherical right angled triangle, given the hypothenuse and the sum of the sides to determine the triangle.

8. To determine when a given error in the observation of an angle produces the least possible error in the computation of its cosine.

9. If two right angled triangles have a common acute angle, the sines of the angles not common are inversely, as the cosines of the angles opposite to the common side.

10. The product of the sines of two angles is to the radius squared, as the sine of their sum to the sum of their co-tangents.

11. The product of the cosines of two angles, multiplied by the difference of their tangents, is equal to the sine of their difference.

12. The product of the tangents of two angles is equal to the difference of the tangents divided by the difference of the co-tangents.

13. Solve the equation

$$x^2 - px = q$$

by the angle whose tangent is $\frac{2\sqrt{q}}{p}$.

14. If $a$, $b$, $c$, be the sides of a spherical triangle, and $A$, $B$, $C$, the angles,

$$\frac{\text{sin. } B}{\text{tan. } A} = \text{sin. } c \text{ cot. } a - \text{cos. } c \text{ cos. } B.$$

15. Also

$$\text{sin. } b = \text{tan. } c \,(\text{sin. } A \text{ cot. } C + \text{cos. } A \text{ cos. } b.$$

## FIRST EVENING.

### TRIGONOMETRY.

1. In a spherical triangle, given the base, vertical angle, and the sum of the base angles, to determine the triangle.

2. Given the base of a spherical triangle, and the sum of the base angles, what are the limits of the area?

3. Given the difference between every pair of sides of a spherical triangle and one of the angles, to determine the triangle.

4. Prove that half the sum of two sides of a spherical triangle is always of the same affection as half the sum of the opposite angles, and shew the solutions of spherical triangles which this properly renders unequivocal.

5. In the determination of an angle of a spherical triangle by that of its segments made by a

perpendicular on the opposite side, how can it be determined whether the sought angle is the sum or difference of its segments?

6. Solve the equation

$$x^3 - 3ax^2 - 3x + a = 0$$

by the angle whose tangent is $a$.

7. Determine the sum of

$$\sin.(a+d) + \sin.(a+2d) + \sin.(a+3d) \cdots$$

continued to $n$ terms.

8. Determine the sum of

$$\cos.(a+d) + \cos.(a+2d) + \cos.(a+3d) \cdots$$

continued to $n$ terms.

9. Prove the formulæ

$$\cos.^2 a - \tfrac{1}{4} = \sin.(60° + a) \sin.(60° - a)$$
$$\cos.^2 a - \tfrac{1}{4} = \cos.(60° + a) \cos.(60° - a)$$

and explain the use of these formulæ.

10. If $\sin.(x+a) = \sin.(x-a)$ what is $x$?

11. Explain the general principles on which the roots of equations may be found by trigonometry.

12. Prove the series

$$\pi = 6.\left\{\frac{1}{2} + \frac{1}{8}\cdot\frac{1}{1.2.3} + \frac{1}{32}\cdot\frac{3^2}{1.2.3.4.5}\cdot\cdots\right\}$$

13. Prove the series

$$\log.(1+x) = M\left(\frac{x}{1} - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} \cdots\cdots\right)$$

14. Deduce from this the series

$$\log.\frac{1+x}{1-x} = 2M\left(\frac{x}{1} + \frac{x^3}{3} + \frac{x^5}{5} \cdots\cdots\right)$$

15. What is the logarithm of 0?

16. Prove that $d$ log. $x = \frac{dx}{x}$.

17. What modification is this subject to, if the logarithms be not hyperbolic?

## SECOND MORNING.

### GEOMETRY.

1. Determine the position of the axes of co-ordinates with respect to a line of the second degree, when the product of the variables and their simple dimensions are wanting.

2. What is the nature of a curve of the second degree, and the position of the axes of co-ordinates with respect to it, when it is represented by an equation of the second degree wanting the square of $x$?

3. If the co-efficients of the squares of the variables be equal and have like signs, the curve is an ellipse, the eccentricity of which is a function of the angle under the axes of co-ordinates.

4. If a diameter of a parabola and a secant intersect at a given angle, what line is a fourth proportional to the intercept of the diameter between the secant and the curve, and the two segments of the secant?

5. What is the limit of the length of the normal in the hyperbola?

6. In the ellipse determine the most oblique system of conjugate diameters.

7. The mean proportional between the perpendiculars from the foci on a tangent is an invariable length.

8. A right line being drawn from the centre meeting the tangent, parallel to the line joining the focus with the point of contact, to find the locus of the point of intersection of this line with the tangent.

9. To determine the locus of the intersection of a perpendicular from the focus with a tangent.

10. Is the property of re-producing itself in its caustic peculiar to the cycloid?

11. How is the area of a cycloid trisected by the generating circle and the curve of sines?

12. Shew that the radius of the osculating circle is always a tangent to the evolute.

13. What are the limits of the magnitude of the normal in the cycloid?

14. How is a multiple point determined?

15. Explain what is meant by a conjugate point, and shew the circumstance from which it derives its existence.

16. Shew that the quadrature of the ellipse depends on that of the circle, and that the quadrature of the hyperbola depends on logarithms.

17. Whence the term *hyperbolic* logarithm?

18. To find the locus of a point from which tangents drawn to a given hyperbola shall have a given difference.

19. Given the vertex of a parabola and the position of a tangent through another point, to determine the locus of the focus.

20. Given in position the centre of an ellipse, in magnitude its greater axis, and a point through which it passes, to determine the locus of the foci.

21. Given the base of a triangle, the difference of the sides, and the difference of the opposite angles, to construct the triangle by the intersection of two loci.

22. Given the base of a triangle, the sum of the sides, and the difference of the tangents of the base angles, to construct the triangle by the intersection of two loci.

23. Determine the position of the centre and axes of an ellipse represented by the equation

$$Ay^2 + Cx^2 + Dy + Ex + F = 0$$

in which all the co-efficients are supposed positive.

24. Shew that this equation cannot represent an ellipse, unless $A$ and $C$ have the same sign.

25. Determine the effect produced on the position of the centre, by the supposition that $D$ and $E$ are negative.

26. Determine the position of the centre if $D$ and $E = 0$.

27. Determine the position of the origin of co-ordinates with respect to this ellipse when $F=0$.

28. Shew that the axes of this ellipse must be parallel to the axes of co-ordinates.

29. Two circles being given in magnitude and position in the same plane, to determine the locus of a point from which tangents being drawn, their difference shall be invariable.

30. To inscribe an ellipse in a triangle so as to touch two sides in given points.

31. To inscribe an ellipse of a given area in a triangle so as to touch the base at its middle point.

32. To determine the point in an ellipse at which the angle under the radius vector from the focus and the tangent is a minimum.

33. In an ellipse, given the length of the perpendicular on the tangent from the focus, to determine the point of contact.

34. In an hyperbola, given the perpendicular from the centre on the tangent, to determine the point of contact.

35. In a parabola, given the polar subtangent, the focus being the pole, to determine the point of contact.

36. In a parabola, to determine the locus of the extremity of the polar subtangent, the pole being at the focus.

37. Given the centre of an ellipse, a point

through which it passes, and the position of a tangent at that point, to determine the locus of its vertices.

38. Given in position the centre of an hyperbola and a point through which it passes, and in magnitude its transverse axis, to determine the locus of the foci of the conjugate hyperbola.

39. Given the base of a triangle, the sum of the squares of the sides, and the difference of the base angles, to construct the triangle by the intersection of two loci.

40. Given the base, the difference of the squares of the sides, and the sum of the tangents of the base angles, to construct the triangle by the intersection of two loci.

41. Given the arithmetic and harmonic means, to determine the extremes.

42. Given the arithmetic mean, and the sum of the geometric and harmonic means, to determine the extremes.

43. Given the geometric mean, and the difference of the arithmetic and harmonic means, to determine the extremes.

44. Given the harmonic mean, and the difference of the arithmetic and geometric means, to determine the extremes.

45. Of three lines in harmonic progression, given the first, and the sum of the second and third, to determine the second and third.

46. Of three lines in arithmetical progression, given the mean and the difference of the extremes, to determine the extremes.

## SECOND EVENING.

### PHYSICS.

1. Prove that the angular motion of a planet varies inversely as the square of the distance.

2. What is the eccentric anomaly?

3. Where is the difference of the mean and eccentric anomaly a maximum?

4. Prove that in this case the cosine of the true anomaly is three-fourths of the eccentricity with the sign changed, if the eccentricity be very small.

5. Determine the relation of the difference between the mean and true distance, to the eccentric anomaly.

6. Explain and prove Ward's Analogy.

7. Prove that his principle is true, if the eccentricity be very small.

8. Given the horizontal parallax of the moon at the equator, to find it in any other latitude.

9. Compute the angle of correction at a given latitude.

10. At what latitude is this correction a maximum?

11. Where is the difference of the eccentric and true anomaly a maximum?

12. Determine the effect of the moon's elliptic motion on eclipses, in apogee and perigee.

13. Exhibit the quantities on which the duration of a solar eclipse depends.

14. Determine the sun's longitude when twilight is shortest in a given latitude.

15. Determine the azimuth at which the altitude of a star varies fastest.

16. Compute the effects of the precession of the equinoxes on the right ascension of the sun.

17. Compute the effects of precession on the declination of the sun.

18. Given the form of an equation between the right ascension and declination of an object, to determine its co-efficients by observations.

19. If a body be moved in an ellipse by a force parallel to one of its diameters, what is the law of the force?

20. A sphere filled with a fluid discharges itself through an orifice at the lower extremity of the vertical diameter, to find the positions of the surface of the fluid at equal intervals of time from the moment it begins to flow.

21. A sphere floats between two fluids, its specific gravity being an harmonic mean between the specific gravities of the fluids; to determine the ratio of the segments into which the vertical

diameter will be divided by the common surface of the fluids.

22. If two perfectly elastic bodies move in the same straight line with given uniform velocities, to determine their velocities after impact.

23. If two inelastic bodies impinge on each other with velocities, the quantity and direction of which are given; to determine the direction of their motion after impact.

24. Shew that an absolute vacuum can never be produced by the air pump.

25. The descents of the mercury in the gauge of the air pump after the successive strokes, are in decreasing geometrical progression.

26. Determine the limit of the angle of incidence, which will allow a ray to pass from a denser into a rarer medium.

27. Determine the position of the centre of a thin lens, as a function of the radii of its surfaces.

28. If the form of the equation between the right ascension and declination be not known, how are observations to be used to determine it?

29. To determine the curvature of a meridian at a given latitude.

30. To determine a point in a parabola described by a body round the focus as centre of force, at which the angular velocity increases fastest.

31. Determine the point in an ellipse described

round the focus as centre of force, at which the angular velocity equals the mean angular velocity.

32. Shew that at this point the equation of the centre is a maximum.

33. Determine the mean angular velocity round a fixed centre of force, as a function of the area described in the time, unity, and the whole area.

34. Determine the ratio of the angular velocity at any distance, in a curve described round a fixed centre of force, to the mean angular velocity.

35. Determine the radius of the circle, whose area is to the area of the orbit, as the angular velocity at a given distance is to the mean angular velocity.

36. The distance at which the angular velocity in the orbit is equal to the mean angular velocity, is such, that a circle described with that distance as radius is equal to the area of the curve.

37. Determine in general the point at which the angular acceleration round the centre of force is a maximum.

38. If $A$ and $B$ be two elastic bodies placed in the same straight line, and any velocity, $a$, be communicated to $A$, to determine the magnitude of a body, $X$, placed between $A$ and $B$, such that the velocity communicated to $B$ through $X$ shall be a maximum.

39. Given the lengths of the arms of a lever,

and also the weights attached to them, to determine the angular motion produced in the lever, by the action of the weights.

40. If a circular ring revolve on one of its diameters, determine the ratio of the sum of the centrifugal forces of all its particles, to the centrifugal force of the whole mass of the ring, revolving in a circle with the same radius and period.

41. Determine the velocity of water flowing from an orifice, as a function of the depth of the orifice, measured from the surface of the water.

42. Shew the relation between the angular velocities of the radius vector and the perpendicular on the tangent, and the linear velocities of a body moving in the curve and in a circle at the same distance.

43. In circles described round the same centre of force, if the velocities are proportional to the periods, shew the law of the force.

44. Determine the point in an ellipse, the focus being the centre of force, at which the angular velocity increases fastest.

# 1819.

## APRIL.

### FIRST MORNING.

#### TRIGONOMETRY.

1. GIVEN two sides and the included angle of a plane triangle, to find the segments into which the third side is divided by the bisector of the opposite angle.

2. In the series

$$\text{sin. } ma = m \text{ sin. } a - \frac{m \cdot m^2 - 1}{1 \cdot 2 \cdot 3} \text{sin. } {}^3a + \frac{m \cdot m.^2 - 1 \cdot m^2 - 9}{1 \cdot 2 \cdot 3 \cdot 4 \cdot 5} \text{sin. } {}^5a \cdots\cdots$$

determine a general formula for the $r^{\text{th}}$ term.

3. Find the value of $x$ in the equation

$$2 \text{ sin. } (x \pm a) = \sqrt{3} \text{ cos. } a \pm \text{sin. } a.$$

4. Given the base and vertical angle of a plane triangle, to determine the ratio of the simultaneous variations of the sides.

5. In a plane right angled triangle, given the radius of the inscribed circle, and one of the sides about the right angle, to determine the triangle.

6. Given the hypothenuse of a spherical right angled triangle, and the sum of the acute angles, to determine the triangle.

7. In a plane triangle given the base, and one of the base angles, to determine the ratio of the small variations of the sides.

8. To determine when a given error in the observation of an angle, produces the least possible error in the computation of its sine.

9. To determine the height of a perpendicular object, the base of which is inaccessible.

10. In a plane triangle, given two sides to determine the ratio of the variation of the remaining side, and the simultaneous variations of one of the base angles.

11. Determine the sum of the series

$$\cos.(45° - x) + \cos.(45° - 2x) + \cos.(45° - 3x) \cdots$$

continued to infinity.

12. Express the sum of the series

$$\cos. a + 2^m \cos. 2a + 3^m \cos. 3a \cdots$$

continued to infinity; $m$ being a positive integer.

13. Express the sum of the series

$$\sin. a + 2^m \sin. 2a + 3^m \sin. 3a + \cdots$$

continued to $n$ terms; $m$ being a positive integer.

14. Express the value of

$$\sin. \tfrac{1}{2} a. \sin. \tfrac{1}{4} a. \sin. \tfrac{1}{8} a \cdots$$

continued *ad infinitum*.

15. Express the sides of a plane triangle, as functions of the radius of the circumscribed circle, and the three angles.

## FIRST EVENING.

### TRIGONOMETRY.

1. Determine the value of the angle and side of an equilateral spherical triangle.

2. In an isosceles spherical triangle, the cosine of the base angle is equal to the tangent of half the vertical angle.

3. If $a$, $b$, $c$ be the sides, and $A$, $B$, $C$ the angles of a spherical triangle,

$$\frac{\text{sin. } C}{\text{tan. } B} = \text{sin. } a \text{ cot. } b - \text{cos. } a \text{ cos. } C.$$

4. Also,

$$\text{sin. } a = \text{tan. } b \text{ (sin. } C \text{ cot. } B + \text{cos. } C \text{ cos. } a.)$$

5. In a spherical triangle, given the base, vertical angle, and the sum of the sides, to determine the triangle.

6. In a spherical triangle, given the base, the difference of the sides, and the difference of the base angles, to determine the triangle.

7. In a spherical triangle, given the base and vertical angle, when is the sum of the sides a maximum.

8. Given the perimeter of a spherical triangle, when is its area a maximum?

9. Given the sum of the three angles, and the sum of the three sides of a spherical triangle, determine the limits of its area.

10. Given the difference between every pair of

angles of a triangle, and a side, to determine the triangle.

11. Explain and prove Napier's analogies.

12. Given two sides of a spherical triangle, and an angle opposite one of them, to determine the angle opposite the other.

13. Shew that this solution is equivocal, except when determined by the rule that half the sum of the given sides, and half the sum of the opposite angles, are of the same kind.

14. Given two sides of a spherical triangle, and the angle opposite one of them, to find the included angle, by expressions for the segments of the angle sought, made by the perpendicular on the opposite side.

15. In what cases is this solution equivocal, and in what particular cases can the uncertainty be removed?

16. Given two angles of a spherical triangle, and the side opposite one of them, to determine the side opposite the other.

17. In what cases only is this solution equivocal?

## GEOMETRY.

18. The absolute quantity, in an equation of the second degree, being wanted, determine the position of the origin of co-ordinates.

19. To determine the relation of the axis of $x$ to a curve of the second degree, when the square

of $x$, and its simple dimension, are wanted in the equation.

20. In every equation which represents a parabola, the square of at least one the variables must occur.

21. The rectangles under the segments of secants to a line of the second degree, parallel to two right lines, given in position, are in a given ratio.

22. If a tangent to an hyperbola be produced to meet its asymptotes, the intercepts between the point of contact and each asymptote are equal.

23. Determine the locus of the point of intersection of the perpendicular from the focus of a parabola on the tangent, with the tangent.

24. Express the ratio of the parameters of two diameters of a parabola, as a function of their inclinations to their ordinates.

25. Given the base and vertical angle of a triangle, to determine the locus of the intersection of the bisectors of the sides.

26. Given the base, the difference of the base angles, and the vertical angle, to determine the triangle, by the intersection of the hyperbola and the circle.

27. Shew that the hyperbola used in this case must be equilateral.

28. Given the base and vertical angle of a triangle, to determine the locus of the intersection

of perpendiculars to the sides, from the opposite angles.

29. In a given parabola, to determine the point of contact, being given the angle under the tangent, and the diameter passing through the point.

30. To determine the point of contact in an ellipse, being given the angle under the tangent, and a line drawn to the focus.

31. To find the greatest ellipse which can be inscribed in a triangle.

32. Of the inscribed ellipses, the centres of which lie upon the bisector of the base, to determine that which has the greatest area.

33. Amongst the several parabolas which may circumscribe a given triangle, to determine that whose segment, cut off by any side of the triangle, shall be a minimum.

## SECOND MORNING.

1. To inscribe an ellipse in a triangle, touching the base at the middle point, and so that the line joining the points of contact with the sides, shall be of a given length.

2. To determine the ellipse inscribed in a triangle, and touching one side at its middle point, which shall have its area a minimum.

3. To determine the point in an ellipse, from

which the angle under the radii vectores from the foci, is a maximum.

4. In an hyperbola, given the length of the perpendicular from the focus, on the tangent, to determine the point of contact.

5. In an ellipse, given the polar sub-tangent, corresponding to a radius vector from the focus, to determine the point of contact.

6. In an hyperbola, to determine the locus of the extremity of the polar sub-tangent, the centre of the hyperbola being assumed as pole.

7. A perpendicular being drawn from the centre of an ellipse on the tangent, to find the point where the intercept of the tangent between it and the point of contact is a maximum.

8. A point is so assumed in a given angle, that the sum of the perpendiculars on the sides of the angle is invariable, determine its locus.

9. To construct a parabola, touching a right line given in position, having a given point as focus, and having its vertex in another right line given in position; and assign the limits within which the solution is possible.

10. To construct a parabola, touching a right line given in position, having its vertex at a given point, and its focus upon a right line given in position, and shew when the solution is impossible.

11. Define the lituus, and determine the position of its asymptote.

12. Determine the spiral in which the tangent of the angle under the radius vector and tangent, is proportional to the angle under the radius vector and fixed axis.

13. Determine the spiral in which the triangle under the polar sub-tangent and radius vector is constant.

14. The sine of an arc being produced, until the produced part bears a given ratio to the arc, to determine the curve which is the locus of the extremity of the produced part.

15. Determine the evolute of the epicycloid.

16. To determine the locus of a point, from which tangents drawn to a given ellipse, shall have a given ratio.

17. Given one focus of an ellipse, a point through which it passes, and the length of the greater axis, to determine the locus of the vertex.

18. Given the vertex of an hyperbola, a point through which it passes, and the magnitude of the greater axis, to determine the locus of the foci.

19. Given the centre of an ellipse, a point through which it passes, and the position of a tangent at that point, to determine the locus of the foci.

20. Given the arithmetical and geometrical means between two lines, to determine the lines.

21. Given the geometric and harmonic means to determine the lines.

22. Given the arithmetic mean, and the difference of the geometric and harmonic means, to find the extremes.

23. Given the harmonic mean, and the sum of the geometric and arithmetic means, to find the extremes.

24. Of three lines in harmonical progression, given the mean and the sum of the extremes, to determine the extremes.

25. Given the sum of three lines in harmonical progression, and the sum of their squares, to determine them severally.

26. Given the harmonical mean, and the product of the extremes, to determine the extremes.

## PHYSICS.

27. Prove that the sine of elongation of a planet, is to the sine of the difference of the longitudes of the earth and planet, as the tangent of the geo-centric latitude is to the tangent of the helio-centric latitude.

28. Find when a given fixed star moves vertically, by the diurnal motion.

29. Determine the time of year when twilight is shortest in a given latitude.

30. Determine the duration of the shortest twilight.

31. What is meant by the terms "equation of the equinoxes," and "mean equinox?"

32. Explain the method of finding the refraction by observing an altitude, and the transit of the sun or a star, and noting the time between them.

## SECOND EVENING.

1. What limits are there to the position of three reflectors, so placed that a person may see his profile after the third reflection.

2. Determine all the different spirals in which the force directed *to* the pole is inversely, as the third power of the distance.

3. Determine that in which the force directed *from* the pole is inversely as the third power of the distance.

4. Determine the class of curves in which the force, in the direction of the ordinate, varies inversely as the third power of the ordinate.

5. Determine the curve in which the force directed along the ordinate varies, directly as the third power of the ordinate.

6. Determine the velocity of rotation of the earth, in order that the weight of a body at the latitude 45° should be half the weight it would have, were the earth not to revolve.

7. Shew that in a strict mathematical sense the weight of a body at midnight, should be less than its weight at noon.

8. Given that the times of descent from all heights to a centre of force is the same, to determine the law of the variation of the force.

9. The time of falling down a given plane at an elevation of 45° is one second; find the elevation of the same plane which would make the time of descent two seconds.

10. Explain the method of correcting the position of an assumed meridian, by observing the transit of a star of known right ascension south of the zenith, and setting the clock by it.

11. Shew that the error of the clock will in this case be less, the nearer the star is to the zenith.

12. What is the longitude of the sun when the earth is at its mean distance?

13. Compare the moment of inertia of a circle revolving round one of its diameters, with that of an equilateral triangle inscribed in the circle, so that one of its perpendiculars coincides with the axis of rotation.

14. If a body move in an hyperbola, the force acting parallel to an asymptote, what is the variation of the velocity?

15. A sphere of a given specific gravity floats between two fluids, the specific gravity of the lighter of which is given, and the vertical diameter of the sphere is cut in a given ratio by the common surface of the fluids, to determine the specific gravity of the heavier fluid.

16. Shew that the equation of time must vanish between the first point of Aries and that point of the first quadrant of the ecliptic, at which the difference between the longitude and right ascension is a maximum.

17. Determine the law of the force directed to the pole of the spiral represented by the equation

$$z^3 = a^3 \cos. 3\omega,$$

$z$ being the radius vector, and $\omega$ the angle which it makes with a fixed axis.

18. Shew that the equation of time must vanish between the winter solstitial point and perihelion.

19. Explain the different methods of finding perihelion.

20. Determine the value of the true anomaly when the true motion of the sun is equal to its mean motion.

21. What is the method of finding a planet's nodes?

22. If two perfectly elastic bodies, $A$ and $B$, move in the same straight line with the velocities $a$ and $b$, to determine the velocity gained by the one and that lost by the other after impact, and shew their common ratio to the velocities which would have been gained or lost if the bodies were perfectly inelastic.

23. If a perfectly hard body impinges with a given velocity and at a given angle upon a per-

fectly hard plane, to determine the velocity it loses.

24. The quantities of air which remain in the receiver after every stroke of the pump, are in decreasing geometrical progression.

25. Determine the ratio of the burning powers of two convex lenses.

26. Describe the experiments by which Newton established the heterogeneous qualities of the solar rays.

# OCTOBER.

## FIRST MORNING.

### TRIGONOMETRY.

1. PROVE that the sum of the co-tangent and co-secant of double an angle is equal to the co-tangent of the angle itself.

2. Prove that the product of the sine of the sum and cosine of the difference of two angles, is equal to half the sum of the sines of their doubles.

3. What relation does the sum of the tangent and co-tangent of an arc bear to double that arc?

4. In the formulæ,

$$\text{tan. } \tfrac{1}{2}a = \pm\sqrt{\frac{1-\text{cos. } a}{1+\text{cos. } a}},$$

$$\text{tan. } \tfrac{1}{2}a = \frac{\text{sin. } a}{1+\text{cos. } a}.$$

5. Why does the first assume a double sign, and the second not?

6. Determine the sum of the series

$\text{cos. } (45^\circ - x) + \text{cos. } (45^\circ - 2x) + \text{cos. } (45^\circ - 3x) \cdots$

continued to $n$ terms.

7. Determine the sum of the series

$$\text{cos. } a + 2^m \text{ cos. } 2a + 3^m \text{ cos. } 3a + \cdots$$

continued to $n$ terms, $m$ being a positive integer.

8. Express the sum of the series

$$\sin. a + 2^m \sin. 2a + 3^m \sin. 3a \cdots$$

continued *ad infinitum*.

9. Express the value of

$$\cos. \tfrac{1}{2} a \cos. \tfrac{1}{4} a \cos. \tfrac{1}{8} a \cdots$$

continued *ad infinitum*.

10. Express the area of a plane triangle in terms of the radius of the circumscribed circle and the three angles.

11. Given the base of a plane triangle and one of the base angles, to determine the ratio between the variation of the side opposite the given angle, and the corresponding variation of the other base angle.

12. Prove the series

$$\pm 2^{m-1} \sin. {}^m a = \sin. ma - m \sin. (m-2)\, a + \frac{m \,.\, m-1}{1 \,.\, 2} \sin. (m-4)\, a \cdots\cdots$$

and determine its number of terms.

13. In a plane triangle, given a side, the angle opposed to it, and the sum of the remaining sides, to determine the triangle.

14. In a plane triangle, given the base and vertical angle, when is the sum of the sides a maximum?

15. In a plane right-angled triangle, given an acute angle, and the sum of the sides that include it, to determine the triangle.

## FIRST EVENING.

### TRIGONOMETRY.

1. In the series

$$\pm 2^{m-1} \sin.{}^m a = \cos. ma - m \cos. (m-2)\, a + \frac{m \,.\, m-1}{1 \,.\, 2} \cos. (m-4)\, a \ldots\ldots$$

determine the last term.

2. In a plane triangle, given the base, difference of the sides, and the vertical angle, to determine the triangle.

3. In any isosceles spherical triangle, the sine of the side is equal to the sine of half the base, divided by the sine of half the vertical angle.

4. Determine the value of $x$ in the equation,

$$\tfrac{1}{2} \tan. (x + \tfrac{1}{2} a) + \tfrac{1}{2} \tan. (x - \tfrac{1}{2} a) = \sec. a.$$

5. Determine the value of $x$ in the equation,

$$\text{see } (60° + x) - \text{see } (60° - x) = \tfrac{1}{2}.$$

6. In the series

$$2^{m-1} \cos.{}^m a = \cos. ma + m \cos. (m-2)\, a + \frac{m \,.\, m-1}{1 \,.\, 2} \cos. (m-4)\, a \ldots\ldots$$

determine the number of terms.

7. Given the three angles of a plane triangle, and one side, compute the segments of that side, made by a perpendicular from the opposite angle.

8. In a plane oblique triangle, being given two sides, to determine the ratio of any small variation of the contained angle, to the simultaneous variation of the opposite side.

9. In a plane triangle, two sides being given, to discover when a given error in observing the included angle, produces the least possible error in the computation of one of the base angles.

10. Three points are given upon an horizontal plane; a spectator is placed at a fourth, it is required that he shall determine his position by observing the apparent angular distances of the three given points, the spectator being supposed not to be placed in the same straight line with any two of the given points.

11. From a given eminence to observe the horizontal distance of two objects, situate in an horizontal plane.

12. Through a given point to draw a right line parallel to an inaccessible right line.

13. Given the hypothenuse of a right angled spherical triangle, and the difference of the acute angles, to determine the triangle.

14. If two right angled spherical triangles have a common acute angle, the sine of their hypotheuuses are proportional to the sines of the sides opposed to the common angle.

15. The sine of the sun's declination bears an invariable ratio to the sine of his longitude.

16. Given the distance between two places, whose latitudes are given, to find the difference of their longitudes.

17. Determine the ratio of the variation of the tangent, to the simultaneous variation of the sine.

GEOMETRY.

18. Shew that the equation of a parabola must always involve one of the variables in its first power.

19. Prove that in a common ellipse there are but two systems of conjugate diameters, at right angles.

20. From the polar equation of an ellipse the pole being at the centre, investigate the greatest and least diameters.

21. In which of the conic sections is the subnormal constant.

22. Determine the curve, from every point of which perpendiculars drawn to a given hyperbola, are equal.

23. Given the base and vertical angle of a triangle, to determine the locus of the centre of the inscribed circle.

24. From a point on a right line given in position, to draw two tangents to a given parabola, which shall include a given angle.

25. A right line being given in position, to find the point in it, from which two tangents drawn to a given parabola, shall contain the greatest or least possible angle.

26. Two concentric ellipses are described in the same plane, to find a point in one from which tangents drawn to the other, shall be inclined to

its transverse axis, at angles, the product of whose tangents shall be given, and to assign the cases in which the problem is possible or not.

## SECOND MORNING

1. A perpendicular being drawn from the focus, on a tangent, to determine the point in the ellipse at which the intercept between the point of contact and perpendicular is a maximum.

2. A circle, and two lines touching it, are given in position, to determine the locus of the middle point of the intercept of a third tangent, between the two given tangents.

3. Perpendiculars from a point within a given angle, on its sides, include a given area, to determine the locus of the point from which they are drawn.

4. To construct a parabola, with a given point as vertex, touching a right line given in position, and having a given parameter.

5. A right line, and two points, are given in position, two right lines always pass through the given points, and intercept between them a given magnitude of the right line given in position, to determine the line traced out by their intersection.

6. To find the equation of an ellipse, touching the three sides of a given triangle, and shew how far the curve is indeterminate.

7. In a parabola, given the length of the perpendicular from the focus, on the tangent, to determine the point of contact.

8. In an ellipse, given the perpendicular from the centre, on the tangent, to determine the point of contact.

9. In an hyperbola, given the polar sub-tangent, corresponding to a radius vector, from the focus, to determine the point of contact.

10. In an ellipse, the centre being assumed as the pole, to determine the locus of the extremity of the polar sub-tangent.

11. Determine the greatest polygon, of a given number of sides, which can be inscribed in an ellipse.

12. Determine the ratio of the greatest inscribed polygon, of a given number of sides, to the least circumscribed polygon of the same number of sides.

13. The two diagonals of every parallelogram circumscribed about an ellipse, intersect at the centre, and are conjugate diameters.

14. To inscribe in a given ellipse a parallelogram of a given species.

15. Determine what limits the possibility of the solution of this problem.

16. If four circles respectively touch three sides of a quadrilateral figure, the locus of the centres of these circles will be a circle.

17. If a quadrilateral be inscribed in a circle, and another quadrilateral be formed by drawing tangents to the circle, through the vertices of the angles of the first, the diagonals of the two quadrilaterals will have a common point of intersection.

18. Prove that this theorem can also be extended to quadrilaterals inscribed and circumscribed in the same manner about an ellipse.

19. Express the area of a quadrilateral, as a function of its diagonals, and the angle under them.

20. To determine the locus of a point, from which tangents being drawn to a given ellipse, their difference shall be constant.

21. Given the focus of a parabola and the position of a tangent, to determine the locus of the vertex.

22. Given in position the centre of an hyperbola, and a point through which it passes, and in magnitude its transverse axis, to determine the locus of the vertices of the conjugate hyperbola.

22. Given the base of a triangle, the sum of its sides, and the rectangle under them, to construct the triangle by the intersection of two loci.

23. Given the base of a triangle, the difference of the sides, and the sum of the tangents of the base angles, to construct the triangle, by the intersection of two loci.

24. Of three lines, in continued proportion, given the rectangles under the mean, and each extreme, to determine them severally.

25. Of three lines, in continued proportion, given the sums of the squares of the mean, and each extreme, to determine them severally.

26. Of three lines, in geometrical progression, given the differences between the mean and each extreme, to determine them severally.

27. Of three lines, in geometrical progression, given the mean, and the difference of the extremes, to determine them severally.

28. Of three lines, in geometrical progression, given the difference of the extremes, and the common ratio, to determine them severally.

29. Given the difference of the harmonic and geometric means, and also the difference of the geometric and arithmetic means, to determine the terms severally.

30. Given the sum of the three means, and the sum of their squares, to determine the extremes and means severally.

31. Of three lines, in harmonic progression, given the first, and the difference of the second and third, to find the second and third.

32. Of three lines, in arithmetical progression, given the first, and the rectangle under the second and third, to determine the second and third.

33. In the same case, given the first, and the sum of the squares of the second and third, to determine the second and third.

PHYSICS.

34. If a body be moved in a parabola, by a force acting parallel to the ordinates of a diameter, to determine the variation of the velocity.

35. If a body describe a semi-ellipse, by a force parallel to one of the axes, to determine the time.

36. Given the height of an inclined plane, to find the angle of elevation, so that a weight may draw another equal weight up it in the least time possible.

37. A pendulum descends through a quadrant of a circle, determine the point at which its vertical descent is most rapid.

38. Determine the point at which the vertical descent of a cycloidal pendulum is most rapid.

39. A fluid spouts from an horizontal orifice, in the side of a vessel, how does the distance at which it meets the plane, on which the vessel stands, vary?

40. If a body be moved in an ellipse, by a force acting in a system of parallels, what is the variation of the velocity?

41. A spherical vessel, filled with a fluid, discharges its contents through an orifice at the

lower extremity of its vertical diameter; at what point does the surface of the fluid sink most rapidly?

42. A cone, the specific gravity of which is given, floats between two fluids, the specific gravity of the heavier fluid being given, and the common surface of the fluids bisects the altitude of the cone; determine the specific gravity of the lighter fluid.

43. If the force of gravity at different heights above the surface be the same, the density of the air will be represented by the ordinate of a logarithmic, of which the absciss is the height.

## SECOND EVENING.

1. Determine the sub-tangent of the logarithmic curve, whose absciss is proportional to the height, and ordinate to the density of the atmosphere.

2. Does the sub-tangent of this curve vary, and under what conditions?

3. Determine the law of the pressure of the condensed air, in the air-gun, upon the ball, as it advances through the barrel.

4. Determine the diameter of the least circle into which heterogeneous light is collected by a plano-convex lens.

5. Determine the longitudinal aberration of parallel rays, incident upon a spherical speculum.

6. Determine the lateral aberration of parallel rays upon a spherical speculum.

7. Express the longitudinal aberration of a spherical speculum, as a function of the distance ($d$) of the focus of incident rays from the centre, the radius ($r$), and the aperture ($a$).

8. Determine the ratio of the axes of the elliptical speculum in Newton's telescope.

9. Why is the prism so placed that the rays pass parallel to the base?

10. Given the point of projection and the direction and velocity, to determine the equation of the path of the projectile, by the integration of the differential equations.

11. Shew the value of the arbitrary constants introduced in these integrations.

12. In circles described round the same centre of force, determine the law of the force when the velocity varies as the distance.

13. Determine the point at which the vertical velocity of the extremity of a circular pendulum is a maximum.

14. Given the angular distance of a circular pendulum from the vertical line, and also the angular distance at which the vibration begins, to determine the vertical velocity of its extremity.

15. A cannon, placed horizontally, is discharged from the top of a tower, express the distance

from the foot of the tower at which it will meet the horizon, as a function of the strength of the charge, and the height of the tower.

16. If two imperfectly elastic bodies, in which the ratio of the elasticity to the compressing force is given, move in the same straight line with given velocities, to find their velocities after impact.

17. If two perfectly elastic bodies impinge on each other with motions given in quantity and direction, to determine the directions of their motions after impact.

18. Determine the criterion under which a body will slide or roll, friction being taken into account.

19. How does the magnitude of the image of the sun, formed by rays passing through a small circular aperture, and received upon a skreen behind it, vary?

20. The position of the touch-hole of a cannon is given, in a given latitude, to construct a burning glass which shall discharge the cannon every day at noon.

21. Equal elastic bodies, by direct impact, interchange their velocities.

22. What is the resultant of two forces, which are equal and parallel, and in contrary directions, but not in the same straight line?

23. A weight being moved by a given power up an inclined plane, to determine the rate of its

vertical ascent, as a function of the elevation of the plane.

24. Determine the ratio of the times in which two cylindrical vessels will empty themselves through orifices in their bases.

25. Explain the principle and uses of the common syphon.

26. Apply this to explain the phenomena of intermitting springs.

# 1820.

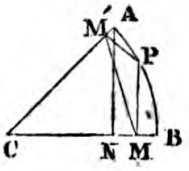
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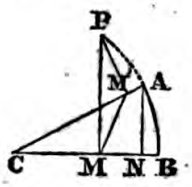
### FIRST MORNING.

#### TRIGONOMETRY.

1. PROVE that half the sum of the sines of two angles is a geometrical mean, between the sine of half their sum and the cosine of half their difference.

2. If $BCA$ be the sector of a circle, and any point, $P$ being assumed in the arc, or produced




arc, perpendiculars $PM$, $PM'$ to the radii be drawn; the line joining the points $M$, $M'$ where these perpendiculars meet the radii is equal to the perpendicular $AN$, from one extremity of the arc, upon the radius passing through the other. Prove this by the formulæ of Trigonometry.

3. Prove the same theorem by Geometry, and deduce from it the fundamental formulæ of Trigonometry.

4. Express the secant of the sum or difference of two angles, in terms of their secants and tangents.

5. If the sum of the tangents of three angles be equal to their product, what is the sum of the angles?

6. What relation does the difference of the tangent and co-tangent of an arc bear to double that arc?

7. In the formulæ

$$\sin.\tfrac{1}{2}a = \pm\sqrt{\frac{1-\cos.a}{2}},$$

$$\cos.\tfrac{1}{2}a = \pm\sqrt{\frac{1+\cos.a}{2}},$$

what is the signification of the double sign?

8. Prove that the sum of the co-tangent and co-secant of an angle is equal to the co-tangent of half the angle.

9. Determine the value of $x$ in the equation,

$$\sin.(x+a) + \sin.(x-a) = \sqrt{2}\cos.a.$$

10. In the series

$$2\cos.ma = (2\cos.a)^m - m(2\cos.a)^{m-2} + \frac{m.m-3}{1.2}(2\cos.a)^{m-4}\ldots\ldots$$

exhibit a general formulæ for the $r^{\text{th}}$ term.

11. Prove the equation

$$\frac{\sin.a + \sin.2a\cdots\cdots\sin.na}{\cos.a + \cos.2a\cdots\cdots\cos.na} = \tan.\frac{n+1}{2}a$$

12. In a plane triangle given two sides and the difference of the opposite angles, to determine the triangle.

13. Prove that

$$\left\{\frac{1+\sqrt{-1}\ \text{tan.}\ a}{1-\sqrt{-1}\ \text{tan.}\ a}\right\}^m = \frac{1+\sqrt{-1}\ \text{tan.}\ ma}{1-\sqrt{-1}\ \text{tan.}\ ma}.$$

14. Find the sum of the series

$$1+\frac{1}{2}-\frac{1}{2.4}+\frac{1.3}{2.4.6}-\frac{1.3.5}{2.4.6.8}+\frac{1.3.5.7}{2.4.6.8.10}\ldots$$

continued *ad infinitum*.

15. Given two sides, and the included angle of a plane triangle, to find the segments of the vertical angle, made by the perpendicular on the third side, by formulæ adapted for logarithms.

16. Given the sum of two angles, determine when the sum of their tangents is a maximum and minimum.

## FIRST EVENING.

### TRIGONOMETRY.

1. In the series

$$\frac{\text{sin.}\ ma}{\text{cos.}\ a} = m\ \text{sin.}\ a - \frac{m.m^2-4}{1.2.3}\ \text{sin.}^3 a + \frac{m.m^2-4.m^2-16}{1.2.3.4.5}\ \text{sin.}^5 a \ldots\ldots$$

give a general formula for the last term.

2. In a plane triangle, given the sum of two sides and the angle opposite to one of them, and the remaining side, to determine the triangle.

3. A river flows by the foot of a tower, to measure its breadth from the opposite bank, by two

observations on the altitude of the top of the tower.

4. Given the base of a plane triangle, and one of the base angles, to determine the ratio of the variation of the side containing the given angle, to the simultaneous variation of the other side.

5. Prove the series

$$\pm 2^{m-1} \sin.^m a = \sin. m a - m \sin. (m-2)\, a + \frac{m \,.\, m-1}{1 \,.\, 2} (m-4)\, a \cdots\cdots$$

and determine its last term.

6. In a plane triangle, given two angles, and the difference of the opposite sides, to determine the triangle.

7. Of all the angles inscribed in the same segment, of which is the sum of the sides the greatest?

8. In a plane right-angled triangle, given an acute angle, and the difference of the sides which include it, to determine the triangle.

9. Given two angles of a plane triangle, and the radius of the inscribed circle, to determine the triangle.

10. Determine the sum of the series

$$\frac{\cos. a}{1} + \frac{\cos. 2a}{2} + \frac{\cos. 3a}{3} \cdots\cdots$$

continued *ad infinitum*.

11. Determine the sum of the series

$$\cos. a + 2 \cos. 2a + 3 \cos. 3a \cdots\cdots$$

continued to $n$ terms $\cdots\cdots$

12. Determine the sum of the series

$$\sin. a + 2 \sin. 2a + 3 \sin. 3a \cdots\cdots$$

continued *ad infinitum*.

13. Find the sum of the series

$$1 - \tan.^2 a + \tan.^4 a - \tan.^6 a \cdots\cdots$$

continued *ad infinitum*.

14. In a plane triangle, given two angles, and the sum of the opposite sides, to determine the triangle.

15. In an isoceles spherical triangle, the sine of the base angle is equal to the cosine of half the vertical angle, divided by the cosine of half the base.

16. Given the three sides of a spherical triangle, to determine the lesser circle which circumscribes it.

## GEOMETRY.

17. To determine the position of the axes of co-ordinates with respect to a line of the second degree, when the first dimension of $y$ does not occur in the equation.

18. To determine the relation which the axis of $y$ bears to a curve of the second degree, when the square and simple dimension of $y$ are wanted in the equation.

19. Find the limits of the sum and difference of a system of conjugate diameters.

20. When is the subnormal in the ellipse a maximum?

21. Two parabolæ have a common axis, to find a point in one, from which two tangents drawn to the other, shall include a given angle.

22. Two tangents drawn to an ellipse from any point on the circle described on the transverse axis, as diameter, include an obtuse angle.

23. To find the point, in the same circle, from which two tangents to the ellipse are inclined at the least possible angle.

24. To determine the point on the asymptote of an hyperbola, from which a tangent to the curve will be perpendicular to the asymptote, and determine the species of hyperbola only in which such a point exists.

25. Define the polar sub-tangent, and give a general formula for it.

26. Determine the polar sub-tangent, corresponding to the radius vector from the focus of an ellipse or hyperbola.

27. Define the hyperbolic spiral, and determine the position of its asymptote.

28. Draw a tangent to the reciprocal spiral, by geometrical construction.

29. Determine the spiral in which the area described, beginning from the pole, is proportional to the radius vector.

30. Determine the spiral in which the polar sub-tangent varies in the sub-duplicate ratio of the angle which the radius vector makes with the fixed axis.

31. Find the equation of an epicycloid, the generating circle of which is equal to the base.

32. Express the equation of this curve, related to rectangular co-ordinates, the origin being at the cusp.

33. Express the equation of the same curve, by polar co-ordinates.

## SECOND MORNING.

1. Given the segment of an ellipse, to determine the greatest angle which can be inscribed in it.

2. The sine of an arc is produced, until the produced part is equal to its chord, to determine the locus of the extremity of the produced part.

3. The polar equation of a curve being given, to give a formula expressing the radius of curvature deriveable from it.

4. Explain the general method of passing from rectangular to polar co-ordinates, and apply this to the investigation of the radius of curvature.

5. Explain the meaning of the evolute of a curve, and the property from which it derives its name.

6. Shew that the evolutes of all algebraic curves are rectifiable.

7. A point is so assumed, within an angle, that the sum of the squares of perpendiculars on the sides shall be invariable, to determine its locus.

8. To construct a parabola, touching a given right line, having a given point as focus, and a parameter of a given magnitude.

9. Given in position a diameter of a parabola, and the point where the curve meets it, and also its parameter, to construct the curve so that its vertex shall be upon a given curve, or straight line, and to shew when the solution is possible.

10. If a pencil be attached to any point in a ruler, and the extremities of the ruler, moved in the legs of a given angle, to determine the nature of the curve traced out by the pencil, and point out the particular case in which this curve is a circle.

11. In a given hyperbola, to determine the point of contact, being given the angle under the tangent, and a line drawn to the centre.

12. To determine the point of contact, in a parabola, being given the angle under the tangent, and a line from the point of contact to the focus.

13. To find the least parallelogram which can be circumscribed round a given ellipse.

14. To inscribe, in a triangle, an ellipse of a given eccentricity.

15. Amongst the angles which may be inscribed in a given parabolic segment, to determine the maximum.

16. Two tangents being drawn to an ellipse, from any point upon a given right line, to determine those which contain the greatest angle.

17. In what respects are the ellipses represented by the equations

$$a^2y^2+b^2x^2-2ab^2x=0$$
$$a^2y^2+b^2x^2+2ab^2x=0,$$

different from one another?

18. Prove analytically that two tangents can, in general, be drawn from a point on the convex side of a line of the second degree.

19. What points are the exceptions to this?

20. Shew that the asymptotes of the hyperbola are the limits which separate the transverse and second diameters.

21. Determine a general criterion by which it may be known whether an equation of the second degree represents right lines, or a curve.

22. Given the base of a triangle, the difference of the base angles, and the rectangle under the sides, to determine the triangle by the intersection of two loci.

23. Determine the magnitude of the surface of the segment of a sphere, cut off by a lesser circle.

24. Determine the magnitude of the solid intercepted between two parallel lesser circles of a sphere.

25. Prove that the solid contents of a sphere equal those of a cone, or pyramid, of which the altitude is the radius, and whose base is equal to the surface of the sphere.

26. Determine the relation between the sphere, its circumscribed cylinder, and its circumscribed

equilateral cone, both as to their solid contents and surfaces.

27. Determine the ratio of the solid contents of a cone and cylinder, of the same base and altitude.

28. What is the difference between a truncated pyramid, and the sum of two pyramids, with altitudes and bases equal to those of the truncated pyramid ?

29. Given the sum of the harmonic and geometric means, and also the sum of the geometric and arithmetic means, to determine the extremes and means severally.

30. Of three lines, in harmonical progression, given the mean and the difference of the extremes, to determine the extremes.

31. Given the arithmetical mean, and the rectangle under the extremes, to determine the extremes themselves.

32. Given one extreme, and the sum of the squares of the harmonic mean and the other extreme, to determine them severally.

33. Given one extreme, and the difference of the squares of the arithmetic mean and the other extreme, to find them severally.

## PHYSICS.

34. If two inelastic bodies, $A$ and $B$, move in the same right line with the velocities $a$ and $b$, determine their common velocity after impact.

35. If a perfectly elastic body impinge upon a perfectly hard plane, to prove the equality of the angles of incidence and reflection.

36. The densities of the air in a condenser, after the successive descents of the piston, are in arithmetical progression.

37. Determine the focal length of a lens, as a function of the radii of its surfaces.

38. In what case will the image of an object, formed by a double convex lens, be equal to the object itself.

39. How does the brilliancy of the image of an object formed by a convex lens vary ?

40. If two perfectly elastic bodies move in the same straight line, with given velocities, the sum of the products of the bodies into the squares of their velocities, before impact, equals the sum of the products of the bodies into the squares of their velocities after impact. *(Conservatio virium vivarum.)*

41. Determine the variation in the length of a degree of latitude, in proceeding from the equator to the pole.

42. A given weight being moved by a given power up an inclined plane, to determine the elevation which shall produce the most rapid vertical ascent.

43. Determine the greatest height to which water can be raised by the common pump.

44. Explain the principle and application of the centrifugal pump.

## SECOND EVENING.

1. A series of elastic bodies being placed in the same straight line, and their magnitudes being in geometrical progression, if the first strikes the second with a given velocity, determine the series of velocities thus communicated to the several bodies of the system.

2. Explain the use of the fusil in a watch.

3. Explain what the centre of percussion is, and the method of determining it.

4. Rejecting the consideration of inertia, friction, &c., determine the space through which a given weight will be raised by a given power, as a function of the time. Also to express the accelerating force of the descent of the power.

5. A vessel being allowed to discharge itself through an orifice at the bottom, to determine the variation of the velocity with which the fluid discharges itself from the bottom.

6. If two perfectly elastic bodies move in the same straight line, with given velocities, the difference of these velocities will be equal to the difference of their velocities after impact.

7. If two imperfectly elastic bodies impinge with motions, given in quantity and direction, to determine the directions and quantities of their motions after impact in functions of their directions, and quantities before impact, and their elasticities.

8. Describe the experiments by which friction is proved to be proportional to the perpendicular pressure, when the nature of the bodies is the same.

9. If the medium through which light passed were perfectly transparent, the distance of an object would not affect its brilliancy.

10. A body floating between two fluids, is bisected by their common surface; determine the relation between the specific gravity of the body and the specific gravities of the two fluids.

11. Determine the ratio of the synchronous areas, in different hyperbolic spirals round the same pole.

12. Given the velocity of projection, and the perpendicular distance of the point of projection from a given plane, to determine the direction of projection, such that the projectile may strike that plane at the greatest possible distance.

13. A ball is shot horizontally from a point one mile above the plane of the horizon, to determine the velocity of projection which will make it describe a circular orbit round the centre of the earth.

14. At what part of the earth's orbit is its angular velocity a geometrical mean between its angular velocities at the apsides?

15. If the square of the periodic time in circles, the force being at their common centre, varies as the radius, determine the law of the force.

16. Prove that if the squares of the periods are as the third powers of the distances, the force must vary as the inverse square of the distance.

17. Determine the curve in which the paracentric velocity varies as the distance.

18. Determine the curve in which the force varies as the chord of curvature, passing through the centre of force.

19. Given the ratio of the velocity in the orbit to the velocity in a circle at the same distance, exhibit the differential whose integral will determine the orbit.

20. If on the radius vector from the focus to any point in a parabola, as chord, a circle be described, and another circle be described with the focus, as centre, and the radius vector as radius, to determine the relation between the velocities of bodies moving in these three curves, at their common point of intersection.

# OCTOBER.

## FIRST MORNING.

### TRIGONOMETRY.

1. Express the sine of an arc as a function of its tangent.

2. Given the hypothenuse and difference of the sides of a right angled spherical triangle, to find the sides themselves.

3. Given an acute angle of a right angled spherical triangle, and the sum of the containing sides, to find these sides themselves.

4. Given the right ascension and declinations of two stars, to find when they will be in the same vertical.

5. Given the three sides of a plane triangle to find the segments into which the vertical angle is divided by the bisector of the base.

6. Secant $(a) = 1 + \tan.(a) \times \tan.\left(\frac{a}{2}\right)$.

7. When a subtense of a right angle drawn through a given point within it, is a minimum, to find its inclination.

8. What is meant by an imaginary logarithm?

9. Area of an hyperbolic sector, and its analogy to a circular arc.

10. Exponential expressions for the sine, cosine, and tangent.

11. Deduce from these the common rule for the sine and cosine of the sum or difference of two arcs.

12. Explain and prove Moivre's formulæ.

13. Find the sum

sin. $a$ + sin. $2a$ + sin. $3a$, &c. to infinity.

14. Explain and prove Cotes's Theorem.

15. Find the series for a power of the cosine in terms of multiple arcs.

16. Find the series for the cosine of a multiple arc in terms of the sine and cosine of the simple arc.

## FIRST EVENING.

### TRIGONOMETRY.

1. Find the sine of a multiple arc in terms of sine of the simple arc.

2. Find the roots of

$$x^3 - 49x - 120 = 0,$$

by trigonometry.

3. Find the action of a force varying as sine of the angle described through 180°.

4. Find the sum of tangents of all the half arcs in the quadrant.

5. Find the sum of all the verse sines in the semicircle.

6. Find the value of

$$(\overline{2\pi}^2 - 1)\,(\overline{4\pi}^2 - 1)\ \&c.$$

to infinity.

7. Logarithmic formula for computing the side of a right-angled triangle, the other two being given.

8. Compute the angle $A$, being given the sides $c$ and $b$, when they are nearly equal.

9. Given the angle $A$, and the difference of the squares of the sides $a$ and $b$, find the side $c$.

10. Where a height is determined by the solution of a right-angled triangle, of what kind should it be.

11. Given the elevation of an object of known height, and its distance above an horizontal plane, to find the height of the observer.

12. Given the height of a mountain, and its elevation above the horizon and the intercepted arc of a great circle of the earth, to find the observer's height.

13. Prove that the sum of squares of cosines of the angle made by a line with three rectangular axes of co-ordinates is equal to unity.

14. Given the angle and perpendicular of a spherical equilateral triangle, to find the side.

15. In a spherical right-angled triangle

$$\cos. \tfrac{1}{2}a + \sin. B = \frac{1}{\sqrt{2}}$$

find the triangle.

16. Determine a formula for the solution of a spherical isosceles triangle.

17. The altitudes and azimuths of a balloon being observed from two stations, the distance of which is known, to find 1°. the distances of the

balloon from the two observes, 2°. its perpendicular distance from the plane of the horizon.

18. Determine the points on the horizon at which the balloon, in any given point of its flight, will have the greatest apparent magnitude.

## SECOND MORNING.

### GEOMETRY.

1. To find the locus of the centre of the circle inscribed in the triangle under lines from the foci to a point in the ellipse.

2. If the rectangle under the segments of the tangent, measured from the vertex of one axis, be equal to the square of the semi-conjugate axis, their extremities are in conjugate diameters produced.

3. The rectangles under the co-ordinates of the extremities of conjugate diameters are equal.

4. The intercept of the tangent between the asymptotes equals the conjugate diameter.

5. To draw from the extremity of the axis major a line dividing the area of the semi-ellipse in a given ratio.

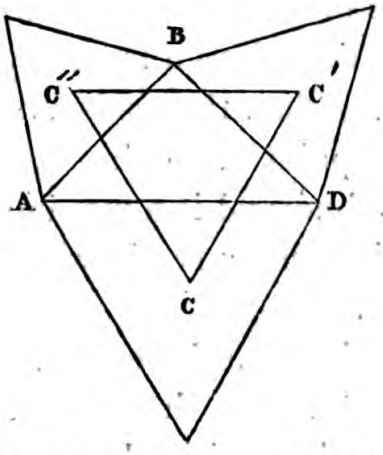
6. Drawing radii of the circle on the transverse axis at right angles, and letting fall perpendiculars from their extemities, their intersections with the ellipse are extremities of conjugate diameters.

7. A circle revolving in another of twice the diameter, find the curve traced by a point in the produced radius of the inner circle.

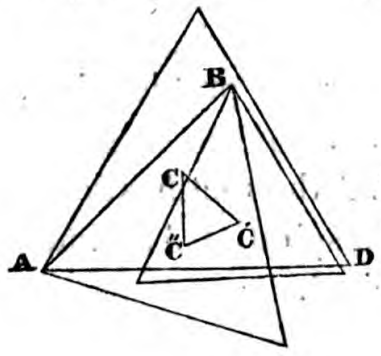
8. Two tangents being drawn to a line of the second degree, and a line drawn joining the points of contact, all secants drawn from the intersection of the tangents will be cut harmonically by the curve and the line joining the points of contact.

9. Three circles are given in magnitude and position, and common tangents are drawn to each pair of them, the three points of intersection of the tangents to each pair of circles lie in the same right line.

10. Three equilateral triangles are thus constructed upon the sides of a given triangle, $A, B, D$,




the lines joining their centres $C, C', C''$, also form an equilateral triangle.



11. If the three equilateral triangles be constructed as in the last figure, the lines joining their centres will also form an equilateral triangle.

12. To investigate the relation between the area of the given triangle and the areas of these two equilateral triangles.

13. To determine the axis and focus of a given parabola by geometrical construction.

14. To determine the centre and axis of an ellipse or hyperbola.

15. Prove by the equation that the sum of any two lines drawn from a point outside the ellipse to the extremities of the transverse axis, is greater, and the sum of lines drawn from any point within the curve is less, than the transverse axis.

16. Find the point in an ellipse, or hyperbola, at which the subnormal is a maximum.

17. Determine the curve in which the subnormal is constant.

18. A parallel to the base of a cycloid being produced until it equals the arc intercepted from the vertex, to determine the locus of its extremity.

19. Trisect an angle by the conchoid of Nichomedes.

20. Solve the same problem by the solution of a cubic equation.

21. Shew that the rectification of the parabola and that of the spiral of Archimedes depend on the same principal.

22. Prove in general that an algebraic curve can only meet a right line in as many points as there are units in the dimension of its equation.

23. To describe a circle passing through a given point, and touching a given ellipse.

24. In an epicycloid whose base and generating circle are equal, there is a certain point through which all lines drawn terminated in the curve are equal. Find this point.

25. Determine the curve in which the sub-tangent bears a given ratio to the abscissa.

26. Explain what is meant by a point of undulation.

27. The curve represented by the equation,

$$y = x^4$$

has a point of undulation, prove this.

## SECOND EVENING.

### PHYSICS.

1. Prove the equable description of areas by integrating the differential equation.

2. Prove the same by the geometrical reasoning of Newton.

3. Prove by integration that if the force varies inversely as the square of the distance from the centre of force, the orbit must be a conic section, of which the centre of force is the focus.

4. Distinguish the cases in which the curve is the ellipse, hyperbola, or parabola.

5. At what point of an ellipse, the centre of force being the focus, is the velocity in it equal to the velocity in a circle at the same distance.

6. Shew from the position of this point that in the parabola, the centre of force being the focus, that the limit of the ratio of the velocity in the curve to the velocity in the equi-distant circle, the distance increasing indefinitely, is a ratio of equality.

7. Determine the limit of the same ratio in the hyperbola.

8. What is the centrifugal force?

9. Shew that if the centrifugal force round a point in any orbit varies inversely as the cube of the distance from that point, it must be the centre of force.

10. In general what relation subsists between that part of the centrifugal force which is directly opposed to the centripetal and the centripetal force itself.

11. Determine the part of the centrifugal force which is in the direction of the normal.

12. Shew in general that the angular velocity varies inversely, as the square of the distance from the centre of force.

13. How does the paracentric velocity vary in general?

14. Find a general formula for determining where the paracentric velocity is a maximum.

15. Apply this to the ellipse when the centre of force is at the focus.

16. Apply it to the parabola in the same case.

17. Apply it to the ellipse when the centre of force is at the centre.

18. How does the ratio of the velocity in the curve to the velocity in the equidistant circle depend on the angle of projection?

19. Determine the conic section in which the force varies in the inverse sub-duplicate ratio of the sine of projection.

20. Shew that in concentric ellipses having the common centre as the centre of force, the periodic time is a function of the absolute force.

21. Determine the point in an ellipse having the centre of force in the centre of the curve, at which the angular velocity is equal to the mean angular velocity.

22. Exhibit a general formulæ for the determination of the point in any curve at which the angular velocity equals the mean angular velocity.

23. What are the methods of determining analytically the apsides of an orbit?

24. How are the places of the apsides of the earth's orbit determined by observation on the sun?

25. Why are the intervals between the equinoxes unequal?

**26. What inconvenience in our measures of time arises from the ellipticity of the earth's orbit?**

**27. Does inconvenience arise from any other cause?**

**28. What is the equation of time?**

**29. Explain what is meant by the mean place of the equinoxes?**

**30. If the sun's true longitude exceeds the mean longitude by as much as it exceeds the true right ascension, the common dial shews the mean time.**

**31. At what points of the ecliptic is the equation of time equal to the equation of the centre?**

**32. At what points of the ecliptic is the equation of time equal to the difference between the true longitude and right ascension?**

# 1821.

## APRIL.

### FIRST MORNING.

#### TRIGONOMETRY.

1. Express the cosine of an arc in a series of powers of the arc itself.

2. Express the arc in series of powers of its cosine.

3. Differentiate an arc as a function of its tangent.

4. Differentiate an arc as a function of its secant.

5. Given

$$F(A) = \sin.\,(30° + A) - \sin.\,(30° - A)$$

to determine the form of $F(A)$.

6. Given

$$F(A) = \cot.\,A - \tan.\,A.$$

to determine the form of $F(A)$.

7. Prove De Moivre's formula by integrating

$$\frac{dx}{\sqrt{1-x^2}}.$$

8. Sum of the $n^{\text{th}}$ powers of the tangent and cotangent of an arc.

9. Is it possible to represent $2\cos.\,(a)$ by

$$x + \frac{1}{x}?$$

10. Determine the value of $x$ in terms of $a$ in this case.

11. Sum of cos. $a$ + cos. $2a$, &c.

12. In determining the angle of a right-angled triangle from the sides about the right-angle, what error is produced by a given error in one of the sides?

13. In determining the hypothenuse from a side and opposite angle, what error is produced by a given error in the angle?

14. Express the sine, cosine of half, and sine of half an angle of a triangle in terms of its sides.

15. Given two sides and included angle, find the remaining side by an auxiliary angle.

16. Extend these formulæ to spherical triangles.

## FIRST EVENING.

### TRIGONOMETRY.

1. Find the bisector of the angle of a spherical triangle, as a function of the sides.

2. Express the area of a spherical triangle in terms of its radius and angles.

3. Surface of a spheric segment equals the circle, whose radius is the chord of half the arch of that segment.

4. The sphere is two thirds of the circumscribing cylinder.

5. Find the angles of the triangle formed by connecting the middle points of the sides of a spherical triangle.

6. Determine the sum of the three angles of a spherical triangle in terms of the sides.

7. Resolve the equation $x^5-1=0$ by Cotes's Theorem, and shew that the form of the roots is impossible.

8. Prove by Cotes's Theorem that the equations $x^m \pm a^m = 0$ have as many roots as they have dimensions, and cannot have more.

9. What are the cube roots of unity?

10. Exhibit the different series for the circumference of the circle with the radius unity.

11. Solve the problem for the trisection of an arch by the intersection of two parabolas.

12. In trisecting an angle thus, what are the significations of the three points of intersection?

13. Shew that the trisection of an angle, and the trisection of a ratio, depend on the same principles.

14. As a planet approaches the horizon, shew that the refraction increases faster than the parallax; and assigning the refraction to vary as the tangent of the zenith distance, shew the exact ratio of the simultaneous increments of parallax and refraction.

15. Given the effect of parallax in altitude, find its effect on right ascension and declination.

16. Given the latitude of the place and the sun's declination to find the time and place of sun-rise.

## GEOMETRY.

17. Equation of the locus of the vertices of all triangles on the same base having a given ratio of the sides.

18. Determine the co-ordinates of the intersections of a given right line and circle.

19. Construct a given quadratic equation by the right line and circle.

20. The tangent of the angle contained in a given segment of a circle is constant, deduce this from the equation of the circle.

21. Through how many points can a line of the second degree be described?

22. How many points are necessary to determine a parabola?

23. Determine analytically the conditions under which it will be possible to describe a parabola passing through four given points.

24. Determine analytically the conditions under which it is possible to describe an ellipse or hyperbola passing through five given points.

25. When does an equation of second degree represent a system of intersecting right lines?

26. When does an equation of the second degree represent a system of parallel lines?

27. Find the centre and axes of the curve represented by a given equation of the second degree.

28. Prove that lines of the second degree in general admit but two axes, and parabola but one.

29. Prove that the two axes of an ellipse or hyperbola intersect each other at right angles.

30. Ellipses of given centre and eccentricity touch a given line; find the locus of the extremities of their transverse axes.

## SECOND MORNING.

### GEOMETRY.

1. What restrictions are laid on the description of an ellipse to touch two or three given lines?

2. Find the locus of the centre of an ellipse touching two given right lines at two given points.

3. Describe a parabola with a given focus, and touching two given lines.

4. Describe a parabola with a given focus, and touching a given right line at a given point.

5. Locus of the vertices of parabolas having a given focus and touching a given line.

6. Locus of the focus of parabolas touching a

given right line at a given point, and having a given diameter through that point.

7. Locus of focus of parabolas having a given vertex, and touching a given line.

8. Given the diameter of a parabola and a tangent through its vertex, to find the locus of the vertex.

9. To find the locus of the extremity of equiconjugates in all ellipses on the same transverse axis.

10. A line passing through a point within an angle revolves round it, to find the locus of the middle point of the intercept.

11. Express the general equation of the asymptotes of an hyperbola, the axes of co-ordinates having no particular position.

12. The intercepts of any right line between an hyperbola and its asymptotes are equal.

13. The rectangle under the segments of a line intercepted between the asymptotes and cut by the curve is of a constant magnitude.

14. A tangent intercepted between the asymptotes is bisected at the point of contact.

15. Ratio of the line drawn from any point in the ellipse or hyperbola to the focus, to the perpendicular from the same point on the directrix.

16. Describing an ellipse on the intercept between the directrices of a given ellipse as axis major, and its transverse axis as axis minor, concentric with it, the ordinate of this curve is equal

to the conjugate diameter of the point where it cuts the given ellipse.

17. Locus of isosceles triangles whose bases are in directum and have a common extremity, and their sides opposite to it passing through a given point.

18. Trisect an angle by this locus.

19. Determine the subtangent of a parabola whose equation is $y^n = px$.

20. Determine the area of the same curve.

## PHYSICS.

21. Determine the general formulæ for finding the centre of gravity.

22. What modification do these formulæ undergo when the body is a plane surface?

23. What do they become when the body is a curve line?

24. Find the centre of gravity of a circular arc.

25. Find the centre of gravity of the area of a circular segment.

26. Find the centre of gravity of a segment of a parabola cut off by an ordinate to the axis.

27. Find the centre of gravity of the area of a triangle.

28. Find the centre of gravity of the arc of a cycloid whose middle point is the vertex.

29. Find the centre of gravity of a pyramid.

30. Find the centre of gravity of the segment of a sphere.

## SECOND EVENING.

### PHYSICS.

1. What is the relation between the density of the air and distance from the centre, gravity being as the $n^{th}$ power of the distance from the centre.

2. How is the barometrical measurement of heights corrected for temperature?

3. In the air-gun given the initial bulk and density of the air, the bulk and weight of the ball, and length of the barrel, to find the velocity of the ball.

4. Define the centre of gyration, and exhibit a formula expressing its perpendicular distance from the axis of rotation.

5. What is the principal centre of gyration?

6. Given the principal centre of gyration; find any other.

7. What is the centre of oscillation, and exhibit a formula expressing its perpendicular distance from the axis of rotation.

8. Find the centre of oscillation of a triangle vibrating on an axis passing through its vertex parallel to its base.

9. Find a general formula expressing the moment of inertia of a solid of revolution.

10. Find the inclination of the plane up which a given weight is drawn with the greatest verti-

cal velocity by another given weight descending perpendicularly.

11. Determine the velocity communicated to a given weight at one extremity of a lever by another given weight acting at the other extremity, the lever having revolved from the horizontal position through a given angle.

12. What are differential equations of accelerated motion?

13. Shew in general that the velocity is independent of the species of the trajectory.

14. Prove the equable description of areas as well from the method of Newton as from the differential equations of motion.

# 1822.

## JANUARY.

### FIRST MORNING.

#### TRIGONOMETRY.

1. FIND the value of $x$ in the equation

$$a \cos. x + b \sin. x = 0$$

by trigonometry.

2. Determine the roots of

$$x^4 + 2x^3 - x^2 - 2x + 1 = 0$$

by trigonometry.

3. Find the sum of

$$\left(1-\frac{1}{2^2}\right)\left(1-\frac{1}{2^2.3^2}\right)\left(1-\frac{1}{2^2.5^2}\right) \text{ &c. &c.}$$

continued *ad infinitum*.

4. Find the sum of

$$\left(1-\frac{1}{6^2}\right)\left(1-\frac{1}{2^2.6^2}\right)\left(1-\frac{1}{3^2.6^2}\right) \text{ &c. &c.}$$

continued *ad infinitum*.

5. Find the sum of

$$1-\frac{1}{3}+\frac{1}{5}-\frac{1}{7}, \text{ &c.}$$

continued *ad infinitum*.

6. Find the sum of

$$1+\frac{1}{5}+\frac{1}{9}+\frac{1}{13}, \text{ \&c.}$$

continued *ad infinitum*.

7. A given number, $n$, of equal weights are placed at equal distances on the vertical semicircle of a wheel, none being at the extremities of the vertical diameter, the radius of the wheel is $r$; to find the statical moment of the weights with respect to the centre of the wheel.

8. To divide an arc so that the sine of one part shall be an arithmetical mean between the sines of the other part and of the whole arc.

9. Prove by trigonometry that the cosine of the angle made by two lines is equal to the sum of the products of the cosines of the angles which they make with three rectangular axes of co-ordinates.

10. To determine the length of an arc of a great circle joining the middle points of two sides of a spherical triangle, as a function of the sides.

11. Resolve a small spherical triangle by the methods of plane trigonometry.

12. Find the pole of the parallel circle which circumscribes a given spherical triangle.

13. Given the distance of two points near the horizon; find the correction for reducing it to the horizon.

14. Find the sum of the angles about the hypothenuse of a right-angled spherical triangle.

15. In a spherical triangle it is given that

$$4 \cos. a = \cos. (b-c) + 3 \cos. (b+c)$$

to determine the angle opposite to the side $a$.

16. The altitude of a meteor is observed from three given stations on an horizontal plane to find its real height and position.

17. Determine the meridian altitude from an observation a few minutes off the meridian.

18. Given the sun's diurnal change of declination; find the corresponding change in the length of the day.

## FIRST EVENING.

### TRIGONOMETRY.

1. Given the sun's diurnal change of longitude; find his change of declination.

2. Find the formulæ for the parallax of a fixed star in latitude and longitude.

3. Determine the value of $S$ if

$$S = \cos. A + \cos. 2A + \cos. 3A \cdots\cdots \cos. nA.$$

4. Determine the value of $S$ in the same equation if $n$ be infinite.

5. Determine the value of $S$ if

$$S = \sin. A + \sin. 2A + \sin. 3A \cdots\cdots \sin. nA.$$

6. Determine the value of $S$ in the same equation if $n$ be infinite.

7. Prove De Moivre's formulæ.

8. Prove Vieta's property of chords.

9. If $y$ and $z$ be the sine and cosine of $x$, prove that

$$\sqrt{-1}\, dx = \frac{dz + \sqrt{-1}\, dy}{z + \sqrt{-1}\, .\, y}.$$

10. Deduce from this equation the formulæ

$$\text{cos. } x = \frac{e^{x\sqrt{-1}} + e^{-x\sqrt{-1}}}{2}$$

$$\text{sin. } x = \frac{e^{x\sqrt{-1}} - e^{-x\sqrt{-1}}}{2}$$

11. Deduce from these formulæ the series for the sines and cosines of multiple arcs.

12. Calculate the sine of one minute, and deduce from that the sines of the other parts of the quadrant.

13. Given the hypothenuse and the sum of the sides of a right-angled spherical triangle, to find the sides themselves.

14. In a right-angled spherical triangle, given one of the acute angles and the sum of the opposite side and hypothenuse, to find the side and hypothenuse separately.

15. Given the difference of the longitudes of a place upon the line and one at a given latitude, to determine the distance between them.

16. If two right-angled spherical triangles have

a common hypothenuse, the tangents of their acute angles are reciprocally proportional.

17. Given the latitudes and longitudes of two places, to find the distance between them.

GEOMETRY.

18. Find a third proportional to the sum of subtangent and subnormal, and the conjugate semi-diameter.

19. The rectangle under the intercepts of the normal between the point of contact and the axes is equal to the square of a given line.

20. The rectangle under the normal and perpendicular from the centre on the tangent is equal to the square of a given line.

21. Determine the locus of the middle points of all the normals of the ellipse.

22. Extend a similar investigation to the parabola and hyperbola.

23. A diameter of a circle revolves on an eccentric point, while its middle point carries a plane surface, and its extremities two perpendiculars, always in contact with the circle; a fixed pencil traces on the plane an ellipse—(oval engine of *the lathe.)*

## SECOND MORNING.

### GEOMETRY.

1. Express the perpendicular from the focus on the tangent in terms of $x$ and constants.

2. Ratio of the intercepts of the normal between the point of contact and axes.

3. Given a parabola, to find the point in it whose normal passes through a given point in the tangent at the vertex.

4. Describe a circle with a given centre touching a given parabola.

5. Locus of the points of contact of concentric circles with similar and similarly posited concentric ellipses.

6. Drawing diameters of the parabola cutting the ordinates to a given diameter, so that the intercepts of these diameters between the ordinates and curve shall be a third proportional to the parameter of the given diameter, and the segment of it cut off by the ordinate, to find the locus of the intersection.

7. In the ellipse, when is the intercept of the tangent between the axes produced a minimum?

8. In the parabola producing the ordinate till the whole produced line is equal to normal; find the locus of its extremity.

9. Extend the same investigation to the other lines of the second degree.

10. Inscribe the maximum ellipse in a parallelogram.

11. Producing the ordinate of a parabola till produced part is equal to the distance from the focus, to find the locus of its extremity and determine its position.

12. Extend the same investigation to the ellipse and hyperbola.

13. Express the sector of an ellipse contained by the axis and focal radius vector in terms of the eccentric anomaly.

14. Express the eccentric in terms of the mean anomaly.

15. Find the locus of the extremity of the polar sub-tangent in the logarithmic spiral.

16. Shew that the curvature of this spiral varies inversely, as the radius vector.

17. Find a portion on the tangent of this curve equal to the arc of the curve continued to the pole.

18. Effect the quadrature of this spiral by the formula $\frac{1}{2}\int r^2\, d\omega$.

19. Determine by integration the spiral in which the polar sub-tangent is constant.

20. Prove that the areas described by the radius vector in each successive 180° are as the numbers

$$\frac{1}{0\,.\,1},\ \frac{1}{1\,.\,2},\ \frac{1}{2\,.\,3},\ \frac{1}{3\,.\,4},\ \&c.$$

21. Three pencils are attached to the extremities and an intermediate point, in a ruler of a given length, and the ruler is so moved that the extreme pencils describe right lines given in position to find the line described by the intermediate pencil.

22. If the same ruler be so moved that the extreme pencils describe two circles given in magnitude and position, to determine the line described by the intermediate pencil.

23. Find two mean proportionals by the conchoid of Nicomedes.

24. Trisect an angle by the same curve.

25. Find the evolute of the parabola.

## SECOND EVENING.

### PHYSICS.

1. Expression of the velocity in a trajectory described round a fixed centre of force.

2. What is the constant of this integral?

3. Given $\theta'$ and $r'$ the angle and distance of projection, the force varying inversely as the third power of the distance, find the velocity, such that the orbit may be the hyperbolic spiral.

4. Equation of the spiral thus described.

5. The ratio of the velocity in it to that in a circle at the same distance.

6. The angular velocity in this spiral is equal to the angular velocity in a circle at the same distance.

7. The centrifugal and centripetal forces are equal in it.

8. Shew that the paracentric velocity is uniform, and determine its value.

9. Determine the ratio of the area to the time, and shew that it is the same for all hyperbolic spirals round the same pole.

10. Determine the value of the same ratio in the logarithmic spiral.

11. Determine in this curve the ratio of the velocity to the velocity in a circle at the same distance.

12. Determine the time of the body in this spiral, describing 180°, and shew that the times of describing each 180° are in geometrical progression.

13. Determine the equation of the curve described by a repulsive force parallel and proportional to the ordinates.

14. Determine the equation of the curve described by an attractive force parallel and proportional to the ordinates.

15. Determine the velocity and paracentric velocity in this curve.

16. Determine the value of the centrifugal force in the ellipse.

17. Find the abscissa of the point in the el-

lipse, with the focus as the centre of force, where the velocity is an arithmetical mean between its greatest and least values.

18. Determine the point of the ellipse where the paracentric velocity is a maximum, by differentiation.

19. Given the ratio of the velocity in the ellipse to the velocity in a circle at the same distance to determine the place of the body.

20. To express the velocity in the ellipse as a function of the distance.

21. To express the period as a function of the transverse axis.

22. An horizontal tube, filled with a fluid, revolves round one extremity; find the law of the pressure against the other extremity, arising from the centrifugal force.

23. Supposing it a solid of revolution, find its profile on the supposition that each transverse section is equally pressed.

24. Compare the masses of the central bodies of two planetary systems.

25. The transverse semi-axes, the distance at which the body is projected, and radius of the circle which would be described with the same absolute force and velocity of projection are in harmonic progression.

26. Given the law of the force to determine the velocity acquired in falling in a right line from an infinite distance.

27. Given the law of the force to express the velocity acquired in falling from a quiescent state at the distance $H$ to the distance $D$ from the centre of force.

28. A body falls from the height $H$ in a straight line, and meets a given curve at the distance $D$, having acquired a velocity equal to that in the curve, prove the equation

$$H^n = D^n + n D^{n-1} \tfrac{1}{4} C,$$

where $C$ is the chord of the osculating circle passing through the centre of force, and the force varies as $D^{n-1}$.

29. Jets of water spout from holes at a given depth below the surface of the reservoir at different elevations, locus of their points of greatest elevation.

30. A gun throws its shot a given distance, its elevation is a little increased, and the range is augmented $\frac{1}{111}$th part; the maximum range being known, the change of elevation is required.

31. How would the motion of the earth be affected if the mass of the sun were diminished one half.

32. A body slides down a quadrant of a circle, find the pressure on the curve at any point.

33. A body is projected at right-angles to the line joining two centres of force varying as the distance, find the orbit.

34. The force being inversely as the square of

the distance, and a body descending in a right line towards the centre of force, find the velocity and time?

35. Shew that if the point from which the body begins its descent be the extremity of the axis of a cycloid, the ordinate to the axis through the place of the body represents the time, and the tangent of the angle at which this ordinate is inclined to the curve, represents the velocity acquired.

36. What is the ratio of the times of falling to the centre with this law and the direct law of the distance, the initial force being the same in both cases?

37. What is the ratio of the velocity at any point of the descent to that in a circle at the same distance?

38. What is the equation of a parabola described by a projectile?

39. A jet spouts from a tube in the side of a vessel of fluid, at an elevation of 60°, and strikes the horizontal plane on which the vessel stands at the greatest possible distance, required the place of the aperture?

40. How many quantities are required to determine the orbit described by a body projected from a given point, the force being inversely as the square of the distance?

41. Express the eccentricity of the orbit thus described, and its axis major, in terms of those quantities.

42. If the orbit be a parabola, determine the position of its axis.

43. What is its parameter?

44. Find the curve described by a force parallel to the ordinate, and proportional to $y$.

45. The equation of the catenary is

$$s^2 = x^2 + 2cx$$

$s$ being the arc, and $x$ the corresponding abscissa; required the law of force parallel to $x$, by which it may be described.

46. Determine the law of force in the hyperbolic spiral by the formula of Laplace.—(See *Lardner's Central Forces.)*

47. No spiral ($r^n = A\omega$) can be described by a force $xr^s$, unless $s$ be 3.

48. Express the area described by the radius vector of the hyperbolic spiral in one revolution, as a function of $r$.

49. Throw a projectile at such an elevation that the product of its range and perpendicular impact on the horizontal plane may be a maximum.

# APRIL.

## FIRST MORNING.

### TRIGONOMETRY.

1. FIND the sum of the series

$$\sin. a + \sin. 3a + \sin. 5a \cdots\cdots$$

continued to $n$ terms.

2. Find the sum of the series

$$\sin. a + \sin. (a+h) + \sin. (a+2h)$$

continued to $n$ terms.

3. Find the sum of the series

$$\cos. 2a + \cos. 4a + \cos. 6a \cdots\cdots$$

continued to $n$ terms.

4. Find the sum of the series

$$\cos. a. + \cos. 3a + \cos. 5a \cdots\cdots$$

continued *ad infinitum*.

5. When the difference between the hypothenuse and one side of a plane right angled triangle is very small, it is equal to half the square of the remaining side divided by the hypothenuse.

6. The versed sine of a very small arc is equivalent to half the square of the arc, the radius being unity.

7. The difference between the secant of a very small arc and the radius is half the square of the arc.

8. To determine the height of a vertical object, the base of which is inaccessible.

9. The altitudes of the top of a tower being observed from two stations, the distance between which is known, to determine the height of the tower by observing at each station the angular distance between the foot of the tower and the other station.

10. Determine the sum of the series

$$\frac{\cos. a}{1}+\frac{\cos. 2a}{2}+\frac{\cos. 3a}{3}\ldots\ldots$$

continued to $n$ terms.

11. Determine the sum of the series

$$\cos. a+2\cos. 2a+3\cos. 3a\ldots\ldots$$

continued *ad infinitum*.

12. Determine the sum of the series

$$\sin. a+2\sin. 2a+3\sin. 3a\ldots\ldots$$

continued to $n$ terms.

13. Find the sum of the series

$$\frac{1^2\sin.^2 a}{1.2}+\frac{1^2.3^2\sin.^4 a}{1.2.3.4}+\frac{1^2.3^2.5^2.\sin.^6 a}{1.2.3.4.5.6}\ldots\ldots$$

continued *ad infinitum*.

14. Determine the value of $x$ in the equation

$$\sin.(x+a)-\sin.(x-a)=\sqrt{3}\sin. a.$$

15. In the series

$$2\cos. ma=(2\cos. a)^m-m(2\cos. a)^{m-2}+\frac{m.m-3}{1.2}(2\cos. a)^{m-4}\ldots\ldots$$

determine the last term, and shew the cases in which it is independent of $a$.

16. In a plane triangle given the difference of two sides, the angle opposite one of them, and the remaining side, to determine the triangle.

17. In the series

$$\frac{\text{sin. } m\,a}{\text{cos. } a} = m \text{ sin. } a - \frac{m \,.\, m^2-4}{1 \,.\, 2} \text{ sin. } {}^3a + \frac{m \,.\, m^2-4 \,.\, m^2-16}{1 \,.\, 2 \,.\, 3 \,.\, 4 \,.\, 5} \text{ sin. } {}^5a \ldots\ldots$$

give a general formula for the $r^{\text{th}}$ term.

18. The height of a tower is found by taking its altitude at any assumed distance from its base, to find at what distance a given error in the observation of the altitude will produce the least possible error in computing the height, given the base.

## FIRST EVENING.

1. If two right angled spherical triangles have a common acute angle, the sines of the sides containing the common angle are as the tangents of the sides opposed to it.

2. The product of the cosines of two angles expresses the ratio of the sine of their sum to the sum of their tangents.

3. The sine of the difference of two angles is equal to the product of their sines multiplied by the difference of their co-tangents.

4. Solve the equation

$$x^2+px=q$$

by the angle whose tangent is $\frac{2\sqrt{q}}{p}$.

5. Solve the equation

$$x^3-px+q=0$$

by the angle the sign of which is $\frac{2p^{\frac{1}{2}}}{3^{\frac{3}{2}}q}$.

6. Explain and prove the properties and uses of the supplemental or polar triangle.

7. What are the limits of the sum of the angles of a spherical triangle?

8. What are the limits of the perimeter of a spherical triangle?

GEOMETRY.

9. Given a segment of an hyperbola to determine the greatest angle which can be inscribed in it.

10. The sine of an arc is produced until the produced part is equal to the cosine, to determine the locus of its extremity.

11. Determine the general character of a point of inflection.

12. Give a general formula for the angle under the radius vector of any curve and the tangent.

13. Exhibit a general formula for the radius of curvature corresponding to any point upon a curve related by a given equation to rectangular co-ordinates.

14. Is the property of reproducing itself in its evolute peculiar to the cycloid?

15. In a given ellipse to determine the point of contact, being given the angle under the tangent and a line drawn to the centre.

16. Given in an hyperbola the angle under the tangent, and a line from the point of contact to the focus, to determine the point of contact.

17. To find the greatest parallelogram which can be inscribed in a given ellipse.

18. To inscribe in a triangle an ellipse which shall touch the middle point of the base, and whose centre shall be a given point on the bisector of the base.

19. To circumscribe a triangle by a parabola, the axis of which is parallel to a line given in position.

## SECOND MORNING.

1. If two polygons be circumscribed about the same ellipse, so that the points of contact shall be the middle point of each side, the sum of the squares of the semidiameters passing through the points of contact of the sides of the one polygon shall be equal to the sum of the squares of the semidiameters passing through the points of contact of the sides of the other.

2. If lines be drawn from the centre of the el-

lipse to the several angular points of these polygons, the sum of the squares of the lines drawn to the angles of the one will be equal to the sum of the squares of the lines drawn to the angles of the other.

3. If any polygon be circumscribed about an ellipse, and another be inscribed by the connection of the points of contact, the continued products of the sines of the alternate angles, which the sides of the one polygon make with the sides of the other are equal.

4. To draw two diameters of a conic section which shall have a given ratio and intersect at a given angle.

5. To inscribe in a given ellipse a trapezium similar to a given one.

6. To circumscribe round a given ellipse a parallelogram similar to a given one.

7. Determine the limits to the possibility of the solution of this problem.

8. Given the equation of an algebraic curve, what transformation is necessary in order to reduce one of the axes to coincidence with a rectilinear diameter?

9. Under what condition will all rectilinear diameters of an algebraic curve be parallel to each other?

10. Define the centre of a curve in general.

11. What is the criterion which determines whether a curve has a centre?

12. What is the geometrical signification of the second differential co-efficient?

13. Express the side of a regular pentagon inscribed in the circle whose radius is unity.

14. Express the area of a triangle as a function of the radius of the circumscribed circle and the three angles.

15. Express the perimeter of a triangle as a function of the radius of the circumscribed circle and three angles.

16. If two quadrilaterals have equal diagonals intersecting at equal angles, their areas will be equal.

17. Given an acute angle of a right-angled triangle to determine the hypothenuse in such a manner that the area of the triangle shall be of a given magnitude.

18. To bisect a trapezium by a line parallel to its parallel bases.

19. To divide the area of a triangle into four equal parts by two lines which intersect at right angles.

20. Two points and a circle being given, to find a point in the circumference from which lines being drawn to the two given points and the centre, the sines of the angles under them shall have a given ratio.

21. Express the three perpendiculars as functions of the radius of the circle circumscribed about the triangle and the three angles.

22. A triangle is circumscribed round a circle whose radius is unity, to express its area as a function of its angles.

23. A triangle is inscribed in a circle whose radius is unity, to express the side of a square inscribed in it as a function of the three angles.

24. Given the altitude of a triangle inscribed in a circle whose radius is $R$, and circumscribed about a circle whose radius is $Y$, to find the sides.

25. To express the area of a quadrilateral inscribed in a circle whose radius is unity, as a function of its angles.

## PHYSICS.

26. Determine the points of a planet's elliptic orbit, at which the angular velocity about the alternate focus equals the mean angular velocity.

27. Determine in general the ratio of the angular velocity round the sun to the angular velocity round the empty focus.

28. What relation subsists between the variation of the apparent diameter of the sun and its angular motion in the ecliptic?

29. What is the image of a plane surface in a concave spherical reflector, the plane being inclined at any angle to the axis of the reflector?

30. Find the centre of gravity of the sector of a sphere.

## SECOND EVENING.

1. Explain what the eccentric anomaly is, and shew how it connects the mean and true anomalies?

2. Explain and prove the method of determining the inclination of the orbit of a planet to the ecliptic.

3. How is the mean motion of the moon determined?

4. Why are the solar ecliptic limits greater than the lunar?

5. What is the method of finding the moon's period from ancient eclipses compared with modern observations?

6. Determine the motion of the moon's apsides from observation.

7. What is meant by the terms "evection" and "variation?"

8. To find the right line of quickest descent from a point without a given circle to its circumference.

9. To find in general the right line of swiftest descent from a given point in a vertical plane to a curve described upon the same vertical plane.

10. Extend the property of the synchronism of the chords of a circle terminated in the vertical diameter, to the case where the falling body is retarded by a friction proportional to the pressure.

11. A right angled triangle is so placed that its perpendicular is vertical, and base horizontal, to determine the relation of the sides so that the time of falling down the hypothenuse shall equal the time of falling down the perpendicular and describing the base with the acquired velocity.

12. How are the densities of the satellites of Jupiter compared?

13. A pendulum vibrates seconds on the surface, at what distance from the centre will it vibrate minutes?

14. Why must the gnomon of a dial be inclined at an angle with the horizon equal to the latitude of the place?

15. A dial with a vertical style is so engraved as to shew the apparent time at the summer solstice; at other times of the year of what nature will its error be?

16. At a given latitude a style is inclined to an horizontal plane at a given angle, and in a given vertical, to find the length and position of the shadow on a given day and at a given hour.

17. A solid floats in three fluids, $A_1$, $A_2$, $A_3$, the specific gravities of which are $S_1$, $S_2$, $S_3$, and is specifically lighter than $A_3$, but heavier than $A_1$ and $A_2$, to find the part of the solid which shall be in the fluid $A_1$ and $A_3$.

18. Prove that the altitude of the mercury in the gage of the air-pump can never be equal to the altitude of the barometer.

19. What are the proofs that it is the pressure of the atmosphere which sustains the column of mercury in the barometer?

20. Prove that the densities of the air in the receiver of an air-pump after the successive strokes are in geometrical progression.

21. Determine the caustic by reflection of the logarithmic spiral, the pole of the spiral being the focus of incident rays.

22. Prove that the length of an arc of a caustic by reflection is equal to the excess of the sums of the incident and reflected ray which terminate one extremity above the sum of the incident and reflected ray, which terminate the other extremity.

23. Determine the densities of the rays in the different parts of a caustic by reflection.

24. Shew that the small simultaneous increments of the angles of incidence and refraction are as the tangents of these angles.

25. What is an imaginary caustic?

26. Determine the ratio of the velocity in the curve represented by the equation,

$$z^2 = a^2 \cos. 2\omega$$

to the velocity in a circular orbit at the same distance.

27. Determine in general the ratio of the an-

gular velocity in a curve to the angular velocity in a circle at the same distance, and shew the relation of this to the centripetal and centrifugal forces.

28. Express the centrifugal force as a function of the angular velocity and distance.

29. The velocity in the orbit is to the velocity in an equi-distant circular orbit in the sub-duplicate ratio of the angular velocity of the radius vector to the angular velocity of the perpendicular on tangent.

# OCTOBER.

## FIRST MORNING.

### TRIGONOMETRY.

1. PROVE that in a plane oblique triangle the difference of the sides is to the base as the sine of half the difference of the base angles is to the cosine of half the vertical angle.

2. Prove also that the sum of the sides is to the base as the cosine of half the difference of the base angles is to the sine of half the vertical angle.

3. Prove that in a plane triangle the sum of the sides and base is to the difference between the sum of the sides and the base as unity is to the product of the tangents of the semi-angles at the base.

4. If a right line bisects the vertical angle of a plane triangle, the difference of the segments is to the base as the tangent of half the difference of the base angles is to the tangent of half their sum.

5. Express the value of

$$2^n \sin.\frac{x}{2^n} \cos.\tfrac{1}{2}x \cos.\frac{1}{4}x \cdots\cdots \cos.\frac{x}{2^n}.$$

6. If the dipping-needle be not placed in the plane of the magnetic meridian, determine the relation of the observed dip to the true one.

7. If the dipping-needle play in a magnetic east and west circle, what position will it take?

(In this and the foregoing question the result is independent of the peculiar nature of the magnetic force.)

8. Clear an observed lunar distance from the effects of parallax and refraction.

9. Investigate the effect of refraction upon the time of a star's rising and setting.

10. On the same data compute the sine and cosine of half an angle.

11. What modification must these computations undergo in order to render them applicable to the solution of the problem, given the three angles of a triangle to find a side?

12. In surveying, how is the horizontal angle deduced from the observed oblique angle?

13. What is the rule for the spherical excess?

14. Explain and prove the method of reducing the observed angle to the centre of the station.

15. Investigate the developement of an arc in terms of its tangent.

16. Express an arc in a series of powers of its sine.

17. Express an arc in a series of powers of its cosine.

18. Approximate to the expression for the circumference of a circle whose radius is unity, by means of two arcs, whose tangents are one-half and one-third.

19. Approximate to the value of the circumference by the expression for the arc of thirty degrees in terms of its sine.

20. Determine the value of the difference between the series

$$1+\frac{1}{5}+\frac{1}{9}+\frac{1}{13}, \text{ \&c.}$$

$$\frac{1}{3}+\frac{1}{7}+\frac{1}{11}+\frac{1}{15}, \text{ \&c.}$$

each continued in *infinitum*.

21. Transform the preceding series into a more converging one, and apply it to the discovery of the circumference of the circle.

## FIRST EVENING.

1. Express $\cos.^2 A - \sin.^2 B$ as the product of two trigonometrical lines.

2. Express the ratio of the sum of the tangents of two angles to the difference of the tangents by two trigonometrical lines.

3. To divide a given arc into two parts, such that the sum of their cosines shall be a maximum.

4. To divide a given arc into two parts, such that their sines shall be in a given ratio?

5. If two arcs differ by a small but constant quantity, prove that the difference of their tangents varies inversely, as the square of the cosine of either.

6. Given the three sides of a spherical triangle, compute by logarithms the sine of an angle.

7. Express the value of the cosine of an arc in terms of the tangent of half the arc.

8. Express the sine of an arc as a function of the sine of twice the arc.

9. When is the rectangle under the sine and cosine of an arc a maximum?

10. When is the sum of the sine and cosine of an arc a maximum?

11. Given the sum of two arcs, when is the rectangle under their sines a maximum?

12. Express the difference of the squares of the sines of two angles as the product of two trigonometrical lines.

13. Express the difference between the sine and cosine of an angle as one trigonometrical line.

14. If a given arc be divided into any parts, the sum of their tangents is inversely as the product of their cosines.

15. Shew also that the sum of the co-tangents of the parts is inversely as the product of the sines.

16. Express the difference of the squares of the cosines as the product of two trigonometrical lines.

17. Shew that the tangent of an arc varies quicker than its sine.

18. What does this prove with respect to parallax and refraction?

19. Given three points on the surface of a sphere, determine the lesser circle which passes through them.

## GEOMETRY.

20. Shew that in the language of Analysis the pole of the conchoid is a point of the curve.

21. What is such a point called?

22. Has it any geometrical character?

23. Determine the equation of a tangent to a given point upon the cissoid of Diocles.

24. What is the value for the radius of curvature as a function of the co-ordinates and their differentials, neither of the co-ordinates being considered the independent variable?

25. If in an equation of the second degree the terms involving the square and simple dimensions of one of the variables be wanting, what is inferred respecting the nature of the curve and its position with respect to the axes of co-ordinates?

26. If an equation of the second degree include no dimension of either co-ordinate of the second degree, what is the curve and what is its position with respect to the axes of co-ordinates?

27. Determine the evolute of the common parabola.

28. Express the values of two conjugate semi-diameters of an ellipse or hyperbola, as functions

of any two others, and of the angles under them severally.

29. Determine the ratio of two conjugate semi-diameters in terms merely of their inclinations to two other conjugate semi-diameters given in position.

30. Determine the ratio of the parameters of different diameters of the same parabola in terms of their inclination to their ordinates.

## SECOND MORNING.

1. In the parabola investigate the relation between the angle at which a diameter is inclined to its ordinates and the distance of the diameter from the axis.

2. In the equation

$$a'^2 y^2 + b'^2 x^2 = a'^2 b'^2,$$

by what transformation can the squares of the coordinates be removed and the product introduced?

3. When only is this transformation possible?

4. Prove that in an ellipse the sum of the distances of a point from the foci and in an hyperbola the difference of these distances is independent of the position of the point.

5. Prove that in an ellipse the sum of the squares of any system of conjugate diameters is

equal to the sum of the squares of the axes, and shew how a similar property may be extended to the hyperbola.

6. In general when any properties of an ellipse are deduced from the equation

$$a^2 y^2 + b^2 x^2 = a^2 b^2$$

by what means can we discover the same or analogous properties of the hyperbola.

7. Determine the equation of the asymptotes of an hyperbola, and shew their analogy to the equi-conjugate diameters of an ellipse.

8. Given the first of three lines in continued proportion, and the sum of the other two to determine them severally.

9. Given the sum of the extremes of three lines in continued proportion, and their common ratio, to determine them severally.

10. Of three lines in continued proportion given the sum of the first and second, and also the sum of the second and third, to determine them severally.

11. Of three lines in continued proportion given the sum of their squares and the sum of the lines, to determine them severally.

12. In a decreasing series of lines in geometrical progression, continued *ad infinitum*, given the first term and the sum of the series, to determine the other terms.

13. In a like series given the difference of the

first two terms and the sum of the first term and the sum of the series, to determine the terms severally.

14. If two polygons be circumscribed about the same ellipse, so that their sides shall be all bisected at the points of contact, the sum of the squares of the sides of the one polygon is equal to the sum of the squares of the sides of the other.

15. If two polygons be inscribed by connecting successively the points of contact of the sides of the former polygons, the sum of the squares of the sides of one inscribed polygon is equal to the sum of the squares of the sides of the other.

16. A segment of a conic section being cut off by the ordinate of a diameter, and a polygon being circumscribed about it in such a manner that the parts of the ordinate intercepted between the sides of the polygon and the curve be equal, the continued products of the alternate segments of all the tangents shall be equal.

17. To express the area of the greatest quadrilateral which can be inscribed in a circle whose radius is unity.

18. In every quadrilateral inscribed in a circle the diagonals are as the sums of the rectangles under the conterminous sides.

19. Given the four sides of a quadrilateral inscribed in a circle to find its diagonals.

20. Let $s$ be half the perimeter of a quadrila-

teral inscribed in a circle, and let its sides successively be $c$, $c'$, $c''$, $c'''$; and let $A$ be the angle included by $c$, $c'$, and $A'$ that included by $c''$ $c'''$, then

$$\text{tan. } \tfrac{1}{2} A = \left( \frac{(s-c)\ (s-c')}{(s-c'')\ (s-c''')} \right)^{\frac{1}{2}}$$

$$\text{tan. } \tfrac{1}{2} A' = \left( \frac{(s-c'')\ (s-c''')}{(s-c)\ (s-c')} \right)^{\frac{1}{2}}$$

PHYSICS.

21. Shew that the three equations of equilibrium of forces applied at the same point, are contained in the equation

$$\Sigma .\ S\, \delta s = 0.$$

22. What is the equation of condition in order that any number of given forces should admit of a single resultant?

23. Explain the principles on which this equation is found, and shew in what cases it is inapplicable.

24. What are the forces to which a system can be resolved in case they do not admit a single resultant?

25. Determine the centre of gravity of the arc of a cycloid, whose middle point is the vertex.

26. Determine the centres of oscillation and gyration of the same arcs, the base being the axis of rotation.

27. A bow is drawn by a force of sixty pounds,

applied at the middle of the string, which is thereby drawn twenty inches from its place of rest. An arrow weighing three quarters of an ounce is shot from it, being acted upon by the string through the twenty inches; determine the velocity of projection, the inertia of the bow and string not being taken into account.

28. If the arrow happen to be disengaged from the string at the end of eighteen inches, so as not to be acted on through the last two inches, what will be the velocity of projection?

## SECOND EVENING.

1. Determine in the latter case the time it was acted on by the string, $a''$ being the unit of time.

2. Apply the dynamical principle of D'Alembert to determine the angular velocity of a body round a fixed axis, from the simultaneous impacts of several bodies which adhere to it after the strokes.

3. Investigate the condition under which the stroke must be given in order that the axis of rotation shall suffer no percussion.

4. Of all the systems of axes passing through a given point, shew that the greatest and least moment of inertia belong to two of the principal axes through that point.

5. If $\omega$ be the angular velocity of a body round its axis, in estimating the pressure upon the axis of $z$, why do we find for the component forces parallel to $x$, integrals of the form $\omega^2 \int x\,dm$, whereas in taking them for the purpose of estimating the percussion at the time of the stroke, we have them of the form $\omega \int y\,dm$?

6. Determine the value of $\frac{\overline{QT}^2}{QR}$ (in Newton's formula) for the eccentric circle, the centrical ellipse and the focal ellipse.

7. Determine the ratio of the synchronous areas in different logarithmic spirals, the pole being the centre of force.

8. Determine the periodic times in the same and different logarithmic spirals.

9. Shew that the velocity in this curve is equal to that in the equi-distant centric circle, and that the centripetal force bears an unvarying ratio to the centrifugal force.

10. Determine the horizontal velocity with which a cannon ball should be shot in order to describe a focal ellipse, with the eccentricity of one mile, round the earth.

11. Transform the differential equations of motion referred to three rectangular axes of co-ordinates into three others, where the position is

fixed in space by a line and two angles, scil the curtate distance, the latitude, and the difference between the longitude of the planet and that of its node; and the force is resolved along the curtate distance, perpendicular to it and perpendicular to the ecliptic.

12. Determine the distance from the centre of force at which a circle would be described with the ultimate velocity in the focal hyperbola.

13. Prove that the orbital force of the planets is directed towards the sun, as a consequence of the differential equations of motion and Kepler's first law.

14. Shew that the law of this force is the inverse square of the distance, as a consequence of Kepler's second law.

15. Shew that at equal distances it is the same for different planets, as a consequence of his third law.

16. The sun's force being expressed by $\frac{h}{r^2}$, if we make the arithmetical substitutions for the quantities engaged in the value of $h$, what is the meaning of its resulting numerical value?

17. From the formula

$$Fdr = \frac{c^2\,dr}{r^3} - \frac{c^2}{2}.\ d.\ \frac{dr^2}{r^4\,dv^2}$$

find the value of the force in the logarithmic spiral.

18. Determine the force by the same formula in the circumferential circle and hyperbolic spiral.

19. Determine the ratio of the velocity at any point in a centric ellipse to the velocity acquired by falling to the centre with the force in the curve continued constant.

20. Can this ratio ever be a ratio of equality, and if so, in what case?

21. Explain Prony's method of measuring the distance of the centre of oscillation from the point of suspension in the compound pendulum.

22. Shew that the centres of oscillation and the point of suspension are interchangeable.

23. In an elastic fluid of uniform temperature why cannot the places of two strata of level be transposed and the equilibrium preserved?

24. Why can it not be done, supposing the temperature to vary from one stratum of level to another?

25. Given the moon's horizontal parallax, investigate its effects in latitude and longitude.

26. Prove the law of the conservation of the *vis viva* of a point urged by a force which is directed to a fixed centre, and which varies as the distance.

27. Ratio of the power to the weight in order to produce equilibrium in the endless screw.

28. Given in wheel-work the ratio of the angular velocities of the first and last axles, investigate the common ratio of the number of teeth in the wheel to the number in the pinion, in order that the total number of teeth multiplied by the number of wheels shall be a minimum.

# FELLOWSHIP EXAMINATION,

IN

# MATHEMATICS, PHYSICS, AND ETHICS.

## 1823.

# ADVERTISEMENT.

THE following pages contain the subject of the examination of the candidates for the late vacant Fellowship in the three principal courses, Mathematics, Physics, and Ethics. The questions are detailed as accurately as the circumstances of the examination admitted the reporter to take it down. Some questions were lost in each course, from being inaudible, and some from being imperfectly understood by the reporter. These, however, being very few, the following account may be considered as substantially correct.

TRINITY COLLEGE, DUBLIN,
*June*, 1823.

## FIRST EVENING.

### MATHEMATICS.

1. Two integers which are the least in a given ratio, measure all other integers in the same ratio.

2. If two integers be prime they are the least in their own ratio.

3. If an integer measure a product, and be prime to one factor, it will measure the other factor.

4. If an integer be prime to two others it will be prime to their product.

5. What is an irrational quantity?

6. Give an example in numbers of an irrational quantity.

7. Prove that the square root of two is an irrational quantity.

8. Prove that irrational roots enter an equation by pairs.

9. Prove that an equation, of which the co-efficients are all integers, cannot have a fractional root.

10. Prove that the number of impossible roots in every equation is even.

11. What is the number of roots of an equation determined by?

12. What is the greatest number of impossible roots which an equation of a given degree admits of?

13. Can a cubic equation ever have three impossible roots?

14. Why are the dimensions of the equation of differences lower than those of the given equation?

15. How does the equation of differences point out the existence of equal roots?

16. How are equations between two variables useful in geometry?

17. Define a diameter in general.

18. What is a curvilinear diameter?

19. What is a rectilinear diameter?

20. What is the highest order of diameter which a curve of a given order admits of?

21. What is an absolute diameter?

22. Prove that every diameter of a line of the second order is an absolute diameter.

23. Does an equation of a given order necessarily represent one continued curve?

24. How is it discoverable whether an equation represents one continued curve or a system of curves?

25. Is it necessary that any locus shall represent an equation, and if not, when will an equation have no locus?

26. When will an equation represent a system of unconnected points?

27. How is the number of infinite branches of a curve to be determined from its equation?

28. What is the difference between the parabola and hyperbola?

29. How may it be discovered from the equation of a curve whether one of the axes of co-ordinates be a diameter?

30. Prove that the cosine of the angle under two right lines is equal to the sum of the products of the cosines of the angles which they make with three rectangular axes of co-ordinates respectively.

31. Express the sine of half an angle as a function of the sine of the angle itself.

32. Express the cosine of half an angle as a function of the sine of the angle itself.

33. If twice the cosine of an angle be expressed by the sum of a quantity and its reciprocal, twice the cosine of any multiple of that angle will be expressed by the sum of the powers of the same quantity and its reciprocal, the exponents of which express the multiple of the angle.

34. Determine the value of this quantity.

35. What are the exponential expressions for the sine and cosine of an angle?

36. Determine these by integration.

37. Deduce from them the formulæ of Demoivre.

38. What is the theorem of Cotes?

39. Deduce it from that of Demoivre.

40. Shew the application of Cotes's theorem to the discovery of the impossible roots of numbers.

41. Apply it to find the roots of unity.

42. What is the algebraical method of determining the position of a tangent at a given point of a curve?

43. What is the method of finding the tangent by the differential calculus?

44. Prove by the calculus that any line but the one, thus determined, passing through the given point, must intersect the curve.

45. In the differential calculus do the differentials of the variables necessarily represent infinitely small quantities?

46. Explain the nature of the limits of ratios.

47. What is the common rule for determining the value of fractions, the numerator and denominator of which simultaneously vanish?

48. When does this rule fail?

49. Explain a more general method of discovering the value of such quotes.

50. When will the value be infinite, and when nothing?

51. What is Taylor's theorem?

52. Demonstrate this by the method of Poisson.

53. What is Maclaurin's theorem? Shew that it is included in that of Taylor.

54. Prove Maclaurin's theorem by successive differentiation.

55. What is the criterion for determining a point of inflection?

(Answered, that the second differential co-efficient vanishes.)

56. Explain the principle more generally.

57. What is indicated by the second differential co-efficient becoming infinite?

58. Determine the points of a curve at which the ordinate is a maximum.

59. What analytical character is common to these points, and those at which it is a minimum?

60. What analytical character distinguishes the one from the other?

61. When are such points also points of inflection?

62. What is the general character of a multiple point?

63. What is a cusp, and what species are there of them?

64. Determine by Maclaurin's theorem the number of different ordinates corresponding to the same abscissa.

65. How are the cases, in which two or more become equal, discovered?

66. What is the fundamental principle of the integral calculus?

67. What is the rule for integrating the product of a power of the variable and its differential?

68. How is the value of the arbitrary constant to be determined?

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69. What is the name given to that method of integration which is derived from the form of the differential of a product?

70. Explain and prove the theorem of Bernouilli.

71. What classes of formulæ admit of being integrated by circular arcs?

72. What relation does the side of a decagon inscribed in a circle bear to the radius?

73. What relation does the side of a pentagon bear to the radius?

74. How many regular solids can exist?

75. Why cannot there be more than five?

76. What are the number of sides in the five regular solids respectively?

77. What are the number of solid angles in each of these?

78. Given the value of the plane angle of the side of a regular solid to find the number of solid angles.

79. To determine a circle whose area is equal to the surface of a given cone.

80. To describe a circle which is equal to the conical surface of a truncated cone.

81. To describe a circle which is equal to the whole surface of a truncated cone.

82. Determine the surface of a given sphere intercepted between two parallel planes.

## SECOND MORNING.

### PHYSICS.

1. If three forces produce equilibrium they must be in the same plane.

2. Determine the direction of each as a function of the angle under the directions of the other two.

3. Determine by geometrical construction the resultant of any number of given forces.

4. What is this construction in the case of equilibrium?

5. Laplace's proof by experiments on pendulums that the moving force is proportional to the quantity of matter.

6. How does he infer from this that the accelerating force is as the acquired velocity?

7. Why is the science of equilibrium more abstract than that of motion?

8. By what artifice may several forces in different planes be resolved in the directions of three axes of co-ordinates?

9. Shew that it is necessary to equilibrium that the sums of the resultants in the directions of the three axes should severally equal nothing,

$$X=0,\ Y=0,\ Z=0.$$

10. If a point constrained to move upon any surface moves from one given point of it to another, it will describe the shortest line joining those points.

11. To resolve a given force into three others parallel to it, and acting at three given points.

12. To find the centre of parallel forces when the several forces and their points of application are given.

13. Give an example of this in rerum natura.

14. A heavy body is supported on several fulcra, to determine the several pressures.

15. A heavy plane rests against horizontal and vertical fixed planes, to find the pressure against the vertical fixed plane.

16. What are the four great principles of mechanics?

17. Are the conservation of areas and the motion of the centre of gravity more extensive than these?

18. What is the resultant of several forces not passing through the centre of gravity?

19. Given the ratio of the progressive velocity of a given sphere to the velocity of rotation of a point on its surface, to find the perpendicular distance of the direction of impact from the centre of the sphere.

20. Express in numbers this distance in the case of the earth.

21. On what account is this distance diminished?

22. What property of a sphere facilitates this calculus?

23. What is the analytical definition of the principal axes?

24. What is the mechanical definition of the principal axes?

25. Shew that the principal axes of every plane figure are symmetrical.

26. Determine the moment of inertia of a plane figure round an axis perpendicular to it.

27. Determine the moment of inertia of a plane figure round an axis in its own plane.

28. What are the constants supplied in these integrations?

29. What are the properties of the principal axes?

30. If the *vis viva* vary, what are the conditions of equilibrium?—*(Not distinctly audible.)*

31. Shew that the states of stable and instable equilibrium are alternate.

32. What is the moment of inertia of a parallelopiped round one of its sides?

33. What is its moment round a line parallel to a side passing through the centre of gravity?

34. What is the analytical definition of a radius of gyration?

35. When has a body two systems of principal axes round which the moments are equal?

36. Why cannot the principal radius of gyration be found *immediately* by experiment?

37. What is the method of finding by experiment any other radius of gyration?

38. What is the radius of gyration of a right line?

39. A sphere affected by friction rolls upon an inclined plane; what is the ratio of the spaces it describes to those it would describe on the same plane were it to be free from friction, and not to roll?

40. To find the line of swiftest descent between two given points not placed in the same vertical line.

41. Describe this curve geometrically.

42. What is the diameter of its generating circle as a function of the position of the two given points?

43. A body is moved by the force of gravity on the convex side of the arc of a circle whose plane is vertical, to determine the point at which it will fly off.

44. A body and its axis of suspension being given, what is the distance between the centres of suspension and oscillation?

45. Given the centres of gravity of the several parts of a system to find that of the whole.

46. Given the centres of oscillation of the several parts of a system to find that of the whole.

47. What effect is produced upon the line of vibration of a pendulum by increasing its weight? —*(Not distinctly audible.)*

48. In what does the difficulty of ascertaining the length of the compound pendulum consist?

49. What is Kater's method of finding the length of the pendulum?

50. How is the number of oscillations in twenty-four hours determined.

51. Why is it unnecessary that a pendulum should vibrate on knife edges?—*(Not distinctly audible.)*

52. A pendulous body strikes another quiescent in an horizontal direction, to determine the force of impact.

53. Shew the use of this?

54. In determining the resistance of a fluid, on what does the value of the constant co-efficient depend?

55. Method of determining the path of a projectile in a resisting medium.—*(Not distinctly audible.)*

56. How do the vibrations of a pendulum in a medium whose resistance is as the square of the velocity vary?

57. How is the difference between the lines of vibration of a pendulum in vacuo and one exposed to the atmospheric resistance vary?

58. According to *Biot* what is the best method of weighing.

59. What peculiar qualities in the instrument does this method require?

60. On what does the sensibility of a balance depend, and how does it vary?

61. How does the sensibility of a balance depend on the velocity of its vibration?

62. A balance being loaded with equal weights, determine the angle of vibration, the consideration of the weight of the balance itself being omitted.

63. What is the ratio of the force of gravity at the equator to the centrifugal force?

64. Determine the period of a satellite at the surface from the principles of falling bodies.

65. Determine the variation of the successive periods in a logarithmic spiral.

66. Given the angular velocity of a moveable orbit and the absolute angular velocity to determine the angular velocity in the orbit.

67. Given the angular velocity of the revolving body in the orbit, and also its absolute angular velocity, to determine the angular motion of the orbit.

68. Shew that Newton's method of investigating the motion of the apsides is applicable to eccentric orbits as well as those which are nearly circular.

69. If the resistance of a fluid vary as the compressing force, how does the resistance of its points vary?

70. Determine the height of the atmosphere when supposed homogeneous.

71. What is the difference between the centre of oscillation and the centre of percussion?

72. Determine the locus of the centre of oscillation of a body, the axis of motion being given, and point out the difference between it and the locus of the centre of percussion.

## SECOND EVENING.

### ETHICS.

1. What is the definition of virtue in general, according to Butler?

2. What is the definition of *honestum* according to Cicero?

3. What is its definition according to the Academics?

4. How does Aristotle denominate the moral faculty?

5. How is the word *virtus* in Latin more restricted in its signification than the word virtue in English?

6. How is virtue more extensive in its acceptation with the stoics than with us?

7. What is the reason of this extension?

8. How does it appear from Cicero that the same principle held good with the Peripatetics?

9. Into what species does Aristotle divide virtue in general?

10. Why does he bring these two species under one general head?

11. What does Aristotle declare to be the *subject* of moral virtue?

12. To what subjects in general does Adams apply moral virtue?

13. What subjects in general does Adams exclude from moral virtue, which, however, are generally considered as included under it?

14. What sect of ancient philosophers verbally agree with Adams in this exclusion?—(Answered the Academics.)

15. How may the objections of the Academics be removed by Adams's own principles?

16. Can virtue be properly applied to the Deity?

17. What does virtue signify when applied to actions?

18. What does virtue signify when applied to agents.

19. What is Aristotle's definition of moral virtue?

20. What is Plato's definition of it?

21. How do Aristotle and Plato differ in their ideas of moral virtue?

22. What are the primary *objects* of virtue according to Aristotle?

23. What are Butler's opinions on this?

24. What are the efficient causes of virtue according to Aristotle?

25. What different qualities are required to judge of virtues and of other actions?

26. In what *mean* does Aristotle place virtue?—(Answer in a relative mean.)

27. Why does Aristotle give a certain latitude to this mean?—*(This and some of the preceding questions not distinctly audible.)*

28. What remarkable error pervades Aristotle's system of ethics?

29. What defect does Cumberland attribute to the Aristotelic system?

30. Why are all affections vulgarly called virtues?

31. How does Butler distinguish between affection and virtue in the example of benevolence?

32. Which of the old philosophers reject all affections as virtues?

33. How does Cicero shew that the Stoics held this principle?

32. What is the general cause of perturbations according to the Stoics?

33. Into what classes do the Stoics divide all perturbations?

34. Into what metaphysical error have the Stoics fallen in assigning this cause?

35. What are the species into which Aristotle divides the affections?

36. How does it appear that the Stoics allow no modification of pain?—*(Not distinctly audible.)*

37. What comparison does Butler institute between one of the constancies and the opposite perturbation?

38. The Stoics admit a passive, but reject an

active principle with respect to a certain other affection?

39. Do the Stoics reject pity as an affection?

40. What more noble affection do the Epicureans reject?

41. In what general principle do the Epicureans agree with the Stoics and Peripatetics in the nature of virtue?

42. How do the Epicureans differ from all other philosophers respecting virtue?

43. What are the classes to which Butler reduces the motives to virtue?

44. Why does he omit temperance in the classes of particular virtues?

45. How does Butler prove that self-love should be repressed, although he admits it to be a justifiable feeling?

46. How does Butler differ from the other philosophers as to the superiority of self-love over the other affections?

47. Self-denial is generally, but not always, conducive to virtue.

48. Why do the temptations to vices shew this to be a state of trial?

49. The difficulties of revelation themselves are a species of trial.

50. What analogy shews that obscurity is to be expected in matters of revelation?

51. How is the objection brought against the

obscurity of revelation answered from religion itself?

52. How are those answered who object to the insufficiency and want of clearness in the evidence of revelation, but admit its validity?

53. What reason does Butler assign for his own obscurity?

54. How is it proved from the principles of Hobbes himself that pity is an affection?

55. How does Hutchinson prove that this is erroneous?

56. How far may volition be considered as an excitement?

57. What confusion in other philosophers with respect to the nature of virtue is remarked by Browne?

58. This confusion may also be observed in Browne himself?

59. What is the essential quality of virtue according to Butler?

60. If vice produce certain evil does it follow by the rule of contraries that virtue must produce certain good?

61. By what argument does Browne prove that all the affections equally depend on self-love?

62. How far does he agree with Locke in this?

63. Into what error have both these philosophers fallen on this subject?

64. How does Browne refute the erroneous principle that the will is an affection?

65. In what respect does Hutchinson differ from Adams and Butler as to the nature of virtue?

66. According to Hutchinson, in what ratio does virtue vary?

67. According to Adams to what is virtue proportional?

68. How far is this proportion restricted?

69. Does Hutchinson distinguish between the virtue of the agent and the virtue of the action?

70. Into what error on this subject does he fall in common with the Stoics?

71. Why is more to be considered in the virtue of the agent than in the virtue of the action?

72. Shew the agreement of Adams and Hutchinson as to the influence of rewards and punishments from the meaning of the word obligation?

73. Shew that Warburton agrees in this?

74. How does Cumberland distinguish between the cause of obligation and the love of our own interest?

75. By what simile does he illustrate this distinction?

76. How does Cumberland illustrate the connection?

77. How does Cumberland define virtue?

78. To what origin does Cumberland trace all virtues?

79. What is the logical distinction between

virtues relatively to precedence and justice? How is virtue denominated?

80. In what do all philosophers agree with respect to morals in general?

81. What is the cause of this concurrence?

82. To what does Cumberland ascribe the general ignorance of natural law?

83. How does Hobbes contradict himself in his definition of the liberty of man considered in a state of nature?

84. By what argument is all obligation to natural law, in a state of nature, dissolved?

85. How is this argument refuted?

86. What instance does Cumberland give of Hobbes contradicting his own principles in this?

87. How does Cumberland connect obligation with self-love?—*(Not distinctly audible.)*

88. What is the error of Cumberland as to the nature of evil?

89. Why are the laws of nature more effectually carried into effect by punishments than by rewards?

90. What better reason does Burlemaqui give for this?

91. In a civil state why are causes of complaint (querelæ) more effectually removed by the frequency than by the severity of punishment?

92. What is the error of Cumberland as to the sanction of civil law?

93. Shew that protection cannot be considered as a reward?

94. What is Cumberland's definition of good?

95. In what respect should this definition be restricted?

96. How do Cumberland and Locke differ as to the goodness and desirableness of any thing?

97. What is the difference between the causes of natural evils and the evils of imperfection according to King?

98. What does King consider to be the proper office of the understanding?—*(Officium intellectûs.)*

99. Which are moral or natural evils greater according to King?

100. Prove that the Deity is determined by himself, and not by the goodness of things?

THE END.

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